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APPLICATION OF OPTIMAL CONTROL THEORY
TO LAUNCH VEHICLES
COMPUTER PROGRAM
DOCUMENTATION

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FOREWORD

This report fulfills the reporting requirements for the documentation of the computer optimization program developed under National Aeronautics and Space Administration contract NAS8-21063, Application of Optimal Control Theory to Launch Vehicles.

The research upon which the computer program is based was carried out by the Systems and Research Division of Honeywell Inc. Dr. C. A. Harvey was principal investigator for the study. Mr. Billy G. Davis and Mr. J. R. Redus of the Aero-Astrodynamics Laboratory were the program's contracting officer representatives.

The research program at Honeywell was under the supervision of Dr. Grant B. Skelton. The original computer optimization program was written by Dr. Skelton and the current version of the optimization program was written by Mr. M. D. Ward.

ABSTRACT

This report is a documentation of the digital computer optimization program developed in conjunction with an investigation of the application of optimal control theory to the design of launch vehicle control systems.

The individual subprograms that make up the optimization program are described. The attendant equations which are presented result from the solution of the difference equation formulation of the quadratic control problem and the derived analytical expression for the upper bound on the likelihood of occurrence of mission failure.

The input requirements for two types of computer runs are described (the "optimization iteration" run and the "simplification" run). The expected output for the "optimization iteration" run and the "simplification" run are displayed and identified (through the use of actual output listings).

The "optimization iteration" run and the "simplification" run are discussed. Memory requirements, program modifications due to memory requirements, and the expected running time for the optimization programs are discussed.

The appendices contain flow charts for the entire program, information on data storage and a glossary relating mathematical notation to FORTRAN notation.

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NOMENCLATURE

ϕ	Angular pitch displacement
z	Lateral displacement normal to nominal flight path
η_i	Generalized deflection of i^{th} flexure mode
z_{sj}	Displacement of j^{th} sloshing mass normal to vehicle centerline
β	Gimbal angle
I_{B1}	Bending moment at the fuselage station 36.5 meters forward of the gimbal station
I_{B2}	Bending moment at the fuselage station 66.6 meters forward of the gimbal station
I_{B3}	Bending moment at the fuselage station 80 meters forward of the gimbal station
α	Angle-of-attack
x, ω	Wind filter states
x_1, x_2, x_3	States for distributing wind loads

Other terms not described here are described in Appendix C.

ILLUSTRATIONS

SECTION I

INTRODUCTION

This report is a documentation of the digital computer optimization program developed under contracts NAS8-20155 (Design of a Load-Relief Control System) and NAS8-21063 (Application of Optimal Control Theory to Launch Vehicles). The goal of both of the above studies was the development of a practical design technology based on optimization theory for the launch booster control problem. The booster control problem is caused by wind-induced bending, rotations, and translations. The booster is controlled by gimbaling part of its thrust, the control problem is to design a gimbal controller which maintains bending moments within structural strength limits throughout the flight and produces satisfactory errors in terminal drift, drift rate, and angle-of-attack.

In reference 1 the incident winds and corresponding booster responses were described as random processes, and an event of mission failure was defined. The event of mission failure is that one or more responses exceed preselected limits at burnout or during the flight. The optimization problem is the minimization of an upper bound of the likelihood of the occurrence of the event of mission failure. The problem was formulated in a manner that permitted its solution by means of known optimization theories. The optimization program obtains the solution in the form of a linear, finite-time controller with time-varying gains.

All of the mathematics and theory is presented in references 1 and 2. It will be assumed that the reader of this report is familiar with the material presented in these two references. In addition, it will be assumed that the reader is familiar with FORTRAN.

In Section II a description of the optimization program is presented. The role that each of the subprograms play in the solution of the difference equation formulation of the quadratic control problem is described. Section III gives a complete description of the INPUT requirements and the expected OUTPUT for the program. In Section IV the two types of runs (the "optimization iteration" run and the "simplification" run) that can be made with the program are discussed. Section IV also includes some comments on the future use of the program in terms of possible modifications and expected running times. In Appendix A, flow charts for the entire program are presented. Appendix B describes the storage relationship between the "data array" T(I,J) and the coefficient matrices and vectors which appear in the difference equations. Appendix C relates the mathematical notation of references 1 and 2 with the FORTRAN notation used in the optimization program.

SECTION II

PROGRAM DESCRIPTION

A description of the main program and each of the ten subroutine subprograms is given along with the pertinent equations involved.

Main Program

Four major tasks are performed in the main program. First, the data associated with the differential equations used to describe the missile dynamics, the wind random process, and the distribution of the wind loads is read into the computer in the main program. Second, all of the necessary calculations are performed on this data so that the differential equations can be approximated by difference equations. Third, the quadratic weights are calculated and stored in the main program. Finally, the main program calls the subroutines GAIN, COV and PRB.

Differential Equations

The quadratic theory used to solve the optimization problem requires that the wind-missile system be expressed as first order vector differential equations of the form

$$\dot{x}(t) = F(t)x(t) + G_1(t)u(t) + G_2(t)\bar{v}_\omega(t) + G_3(t)\eta(t) \tag{2.1}$$

and responses to be controlled be expressed as

$$r(t) = H(t)x(t) + D_1(t)u(t) + D_2(t)\bar{v}_\omega(t) \tag{2.2}$$

where $x(t)$ is the state vector with the following 22 components

$$x(t) = [\dot{\phi},\dot{z},\dot{\eta}_1,\dot{\eta}_2,\dot{\eta}_3,\dot{z}_{s1},\dot{z}_{s2},\dot{z}_{s3},\phi,z,\eta_1,\eta_2,\eta_3,z_{s1},z_{s2},z_{s3},\beta,\omega,x,x_1,x_2,x_3]^T,$$

$r(t)$ is a vector of missile response deviations from an ideal (no-wind) trajectory with the following 13 components

$$r(t) = [\beta,\dot{\beta},I_{B1},\dot{I}_{B1},I_{B2},\dot{I}_{B2},I_{B3},\dot{I}_{B3},\alpha,\dot{\phi},\dot{z},\phi,z,]^T ,$$

$u(t)$ is the control input to the actuator, $\bar{v}_\omega(t)$ is the mean incident wind, $\eta(t)$ is the Gaussian white noise input to the wind filter, and the matrices $F(t)$ and $H(t)$ and vectors $G_1(t)$, $G_2(t)$, $G_3(t)$, $D_1(t)$ and $D_2(t)$ are all time varying and of appropriate dimension. The components of $x(t)$ and $r(t)$ are defined in NOMENCLATURE.

Differential Equation Data

The data for the differential equations are the elements of the matrices $F(t)$ and $H(t)$ and the components of the vectors $G_1(t)$, $G_2(t)$, $G_3(t)$, $D_1(t)$ and $D_2(t)$ for the 33 values of time $t = 0, 5, 10, \ldots, 155, 160$ seconds. This data is read into an array T(I,J) from punched cards.

Difference Equations

The simplest difference approximation to equation (2.1) is

$$x[(k+1)\Delta t]=[I+\Delta tF(k\Delta t)]x(k\Delta t)+\Delta tG_1(k\Delta t)u(k\Delta t)+\Delta tG_2(k\Delta t)\bar{v}_\omega(k\Delta t)+\Delta tG_3(k\Delta t)\eta(k\Delta t) \tag{2.3}$$

For a given value of Δt a more accurate approximation to equation (2.1) is given by the sample-data form

$$x[(k+1)\Delta t] = e^{\Delta t F(k\Delta t)} x(k\Delta t) + F^{-1}(k\Delta t)\left[I - e^{-\Delta t F(k\Delta t)}\right]\left[G_1(k\Delta t)u(k\Delta t) + G_2(k\Delta t)\bar{v}_\omega(k\Delta t) + G_3(k\Delta t)\eta(k\Delta t)\right] \tag{2.4}$$

Both of these approximations are used in the program. The sample-data approximations are used for the high frequency dynamics (flexure modes, gimbal deflection, wind states, and states associated with the load distribution) and the simple difference approximation form for the low frequency dynamics of the system.

The necessary calculations are made on the data in the array T(I,J) to implement the approximations to equation (2.1) by equations (2.3) and (2.4). The modified data in T(I,J) is then truncated to four significant figures. (This is to insure that in solving difference equations backward and then forward in time, there will not be a buildup of round-off error.) The 292 truncated difference equation coefficients for $t = 0, 5, 10, \ldots, 155, 160$ seconds are made time-varying coefficients by linear interpolation between the five second data points. The coefficients for $(t=0)$, $(t=160)$ and the coefficient differences for all of the intermediate time intervals are stored in the data array T(I,J) in the following way:

coefficients for $(t=0)$ are stored in	T(1,J)
differences for the interval $0 < t \leq 5$ stored in	T(2,J)
differences for the interval $5 < t \leq 10$ stored in	T(3,J)
differences for the interval $10 < t \leq 15$ stored in	T(4,J)
.	.
.	.
.	.
.	.
differences for the interval $150 < t \leq 155$ stored in	T(32,J)
differences for the interval $155 < t < 160$ stored in	T(33,J)
coefficients for $(t=160)$ are stored in	T(34,J).

Quadratic Weights

The elements of the quadratic weighting matrix $Q(t)$, associated with the in-flight constraints on β, I_{B1}, I_{B2}, and I_{B3}, are calculated for the 33 values of time $(t=0,5,\ldots,155,160$ seconds) and stored in the array FOX(I,J). The quadratic weighting matrix is diagonal. The first eight diagonal elements of $Q(t)$ are the quadratic weights for the first eight components of $r(t)$ (the response vector) β, $\dot{\beta}$, I_{B1}, $\dot{I}_{B1}$, I_{B2}, $\dot{I}_{B2}$, I_{B3}, $\dot{I}_{B3}$. The diagonal elements are stored in FOX as follows:

FOX(I,1) = Q_{11},	FOX(I,7) = Q_{55}
FOX(I,3) = Q_{22},	FOX(I,9) = Q_{66}
FOX(I,4) = Q_{33},	FOX(I,10) = Q_{77}
FOX(I,6) = Q_{44},	FOX(I,12) = Q_{88}

The index I again refers to time. The weights for the initial value of time (t=0), the terminal value of time (t=160) and the weight differences for the intermediate time intervals are stored in the rows of FOX(I,J) in the same way that they are stored in the "data array" T(I,J). The quadratic weights on the terminal responses $\alpha(T)$, $\dot{z}(T)$ and $z(T)$ are Q_9, Q_{11} and Q_{13}. These are the only nonzero elements in Q(N), in equation (2-12), which determines the starting value of the Riccati matrix P(N).

Subroutine Gain

The GAIN subroutine solves a backward difference equation which represents the solution of the difference equation formulation of the Quadratic Control Problem presented in Appendix G of reference 1 and Section III of reference 2. The subroutine is called in the main program with the FORTRAN statement CALL GAIN(II,NS). If II is 1, the scalar deterministic input f(n) and vector g(n) (see equations 2.9 and 2.11) are calculated. If II is 2 f(n) and g(n) are not calculated.† NS is the number of components in the state vector x and is set to 22 at the beginning of the main program.

Quadratic Problem

Suppose the system is given as

$$\begin{aligned} x(n+1) &= A(n)x(n) + B_1u(n) + B_2\bar{v}_\omega(n) + B_3(n)\eta(n) \qquad (2.5)\\ r(n) &= H_1(n)x(n) + D_1(n)u(n) + D_2(n)\bar{v}_\omega(n)\\ m(n) &= H_2(n)x(n) + \xi(n) \end{aligned}$$

where m(n) is a vector of sensor outputs, and $\xi(n)$ is a white noise input,

$$\begin{aligned} &E\{\eta(i)\eta(j)'\} = (\Delta t)^{-1} W_1(i)\delta_{ij},\\ &E\{\xi(i)\xi(j)'\} = (\Delta t)^{-1} W_2(i)\delta_{ij}, \qquad \delta_{ii} = 1,\ \delta_{ij} = 0 \text{ if } i \neq j\\ &E\{\eta(i)\xi(j)'\} = (\Delta t)^{-1} W_3(i)\delta_{ij},\\ &E\{x(0)\} = \bar{x}(0),\ E\{[x(0)-\bar{x}(0)][x(0)-\bar{x}(0)]'\} = x(0), \end{aligned}$$

and $n = 0, 1, \ldots, N$ where $N = (\Delta t)^{-1}T$ (for our case $\Delta t = .02$ seconds and $T = 160$ seconds). The optimization problem is to find the linear transformation of present and past measured responses

$$u(n) = \sum_{i=o}^{n} L(n,i)m(i)$$

That minimizes the quadratic functional

$$J^{**} = TR\{Q(n)[S(N)+R(N)] + \sum_{n=o}^{N-1} \Delta t Q(n)[S(n)+R(n)]\} \qquad (2.6)$$

where TR indicates the trace, S(n) is the response covariance matrix defined by

$$S(n) = E\{[r(n)-\bar{r}(n)][r(n)-\bar{r}(n)]'\},$$

R(n) is the mean-response product matrix cefined by $R(n) = \bar{r}(n)[\bar{r}(n)]'$ and

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† The card in the main program which contains the call statement to the GAIN subroutine must be changed manually if the argument II is to be changed (i.e. the card reads either CALL GAIN (1,NS) or CALL GAIN (2,NS)).

$\bar{r}(n) = E\{r(n)\}$. Assuming that $Q(n)$ is symmetric and nonnegative definite matrix for $n = 0, 1, 2, \ldots, N$, and $[D_1(n)]' Q(n) D_1(n)$ is positive for $n < N$ and $Q(N)D_1(N) = 0$, the solution is:

$$u(n) = K(n)\hat{x}(n) + f(n) \tag{2.7}$$

where $\hat{x}(n)$ is the conditional estimate of the state

$$\hat{x}(n) = E\{x(n)|m(0),\ldots,m(n),u(0),\ldots,u(n-1),\bar{v}_\omega(0),\ldots,\bar{v}_\omega(n-1)\},$$

and $K(n)$ and $f(n)$ satisfy the backward difference equations

$$K(n)=-[B_1'P(n+1)B_1+\Delta tD_1(n)'Q(n)D_1(n)]^{-1}[B_1'P(n+1)A(n)+\Delta tD_1(n)'Q(n)H_1(n)] \tag{2.8}$$

$$f(n)=-[B_1'P(n+1)B_1+\Delta tD_1(n)'Q(n)D_1(n)]^{-1}\{B_1'[g(n+1)+P(n+1)B_2\bar{v}_\omega(n)]+\Delta tD_1(n)'Q(n)D_2(n)\bar{v}_\omega(n)\} \tag{2.9}$$

$$P(n)=[A(n)+B_1K(n)]'P(n+1)[A(n)+B_1K(n)]+\Delta t[H_1(n)+D_1(n)K(n)]'Q(n)[H_1(n)+D_1(n)K(n)] \tag{2.10}$$

$$g(n)=[A(n)+B_1K(n)]'[g(n+1)+P(n+1)(B_1f(n)+B_2\bar{v}_\omega(n)]$$
$$+ \Delta t[H_1(n)+D_1(n)K(n)]'Q(n)[D_2(n)\bar{v}_\omega(n)+D_1(n)f(n)] \tag{2.11}$$

with final (starting) values

$$P(N) = H_1(N)'Q(N)H_1(N) \tag{2.12}$$

and

$$g(N) = H_1(N)'Q(N)D_2(N)\ \bar{v}_\omega(N) \tag{2.13}$$

The GAIN subroutine solves equations (2.8) thru (2.11) with terminal conditions given by (2.12) and (2.13). Values of the gain vector $K(n)$ and the scalar deterministic input $f(n)$ for values of n corresponding to the 161 time points $t = 0, 1, 2, \ldots, 158, 159, 160$ seconds are stored in the array BK(I,J). The dimensions of BK are BK(161,23), where the row index refers to the time points (i.e. $K(t)$ and $f(t)$ for (t=0 seconds) are stored in row 1, for (t=1 second) are stored in row 2 etc.) and the column index indicates the components of the gain vector ($f(t)$ is stored in column 23). After the calculations are concluded and the array BK(I,J) has been printed and punched on cards, the gains and deterministic input are made time varying by linear interpolation between the one second data points. The gains and deterministic input for (t=0 seconds) and the differences for all of the intermediate time intervals are stored in the array BK(I,J) in the following way:

$K(t)$ and $f(t)$	for (t=0 seconds) are stored in	BK(1,J)
Differences of $K(t)$ and $f(t)$ for the interval	$0 < t \le 1$ seconds are stored in	BK(2,J)
Differences of $K(t)$ and $f(t)$ for the interval	$1 < t \le 2$ seconds are stored in	BK(3,J)
.	.	.
.	.	.
.	.	.
.	.	.
Differences of $K(t)$ and $f(t)$ for the interval	$158<t\le159$ seconds are stored in	BK(160,J)
Differences of $K(t)$ and $f(t)$ for the interval	$159<t\le160$ seconds are stored in	BK(161,J).

Subroutine COV

The subroutine COV solves a forward difference equation for the state covariance matrix $X(n)$ for $(n=0,1,\ldots,N)$ and calculates the response covariance matrix $S(n)$ and the mean-response vector $\bar{r}(n)$ for values of n corresponding to the 33 values of time $(t=0,5,10,\ldots,155,160$ seconds). The subroutine is called in the main program with the FORTRAN statement CALL COV(II,NS). If II is 1, mean-responses are calculated. If II is 2, mean-responses are not calculated.† As in the GAIN subroutine, NS is the number of components in the state vector. The state covariance matrix

$$X(n) = E\{[x(n)-E\{x(n)\}][x(n)-E\{x(n)\}]'\},$$

satisfies the difference equation

$$X(n+1) = [A(n)+B_1K(n)]\ X(n)[A(n)+B_1K(n)]' + (\Delta t)^{-1}B_3(n)W_1(n)B_3(n)' \qquad (2.14)$$

The response covariance matrix is

$$S(n) = [H_1(n)+D_1(n)K(n)]\ X(n)[H_1(n)+D_1(n)K(n)]' \qquad (2.15)$$

The mean-response vector is

$$\bar{r}(n) = H_1(n)\bar{x}(n) + D_1(n)[K(n)\bar{x}(n)+f(n)] + D_2(n)\bar{v}_\omega(n) \qquad (2.16)$$

where

$$\bar{x}(n+1) = A(n)\bar{x}(n) + B_1[K(n)\bar{x}(n)+f(n)] + B_2(n)\bar{v}_\omega(n).$$

The response covariances are stored in the three dimensional array RR(I,J,K). The index I runs from 1 to 33 and corresponds to the 33 values of time $(t=0,5,\ldots,155,160$ seconds) for which specified elements of the response covariance matrix are calculated. The response covariances are stored in the array RR in the following way:

$\sigma_\beta{}^2$ = RR(I,1,1)	$\sigma_{I_{B2}}{}^2$ = RR(I,5,5)
$\rho_{\beta,\dot\beta}\sigma_\beta\sigma_{\dot\beta}$ = RR(I,1,2)	$\rho_{I_{B2},\dot I_{B2}}\sigma_{I_{B2}}\sigma_{\dot I_{B2}}$ = RR(I,5,6)
$\sigma_{\dot\beta}{}^2$ = RR(I,2,2)	$\sigma_{\dot I_{B2}}{}^2$ = RR(I,6,6)
$\sigma_{I_{B1}}{}^2$ = RR(I,3,3)	$\sigma_{I_{B3}}{}^2$ = RR(I,7,7)
$\rho_{I_{B1},\dot I_{B1}}\sigma_{I_{B1}}\sigma_{\dot I_{B1}}$ = RR(I,3,4)	$\rho_{I_{B3},\dot I_{B3}}\sigma_{I_{B3}}\sigma_{\dot I_{B3}}$ = RR(I,7,8)
$\sigma_{\dot I_{B1}}{}^2$ = RR(I,4,4)	$\sigma_{\dot I_{B3}}{}^2$ = RR(I,8,8)
$\sigma_\alpha{}^2$ = RR(I,3,1)	$\sigma_\phi{}^2$ = RR(I,1,4)

† The card in the main program which contains the call statement to the COV subroutine must be changed manually if the argument II is to be changed (i.e. the card reads either CALL COV (1,NS) or CALL COV (2,NS)).

$$\left[\sigma_{\dot{\phi}}\right]^2 = RR(I,1,3) \qquad \sigma_z^{\;2} = RR(I,2,4)$$

$$\sigma_{\dot{z}}^{\;2} = RR(I,2,3)$$

where ρ is the correlation coefficient and σ is the standard deviation.

The mean-responses are stored in the array R(I,J). Again the index I corresponds to the 33 values of time (t=0,5,...,155,160 seconds) and the index J indicates the component of the mean response vector. The mean responses are stored in the array R in the following way:

$R(I,1) = \bar{\beta}$	$R(I,8) = \bar{\dot{I}}_{B3}$
$R(I,2) = \bar{\dot{\beta}}$	$R(I,9) = \bar{\alpha}$
$R(I,3) = \bar{I}_{B1}$	$R(I,10) = \bar{\dot{\phi}}$
$R(I,4) = \bar{\dot{I}}_{B1}$	$R(I,11) = \bar{\dot{z}}$
$R(I,5) = \bar{I}_{B2}$	$R(I,12) = \bar{\phi}$
$R(I,6) = \bar{\dot{I}}_{B2}$	$R(I,13) = \bar{z}$
$R(I,7) = \bar{I}_{B3}$	

Subroutine Start

The subroutine START is called by the subroutine GAIN and subroutine COV. Its function is to initialize the coefficient matrices and vectors which appear in the difference equations. For the backward difference equations in the subroutine GAIN, the call statement CALL START (2,NS) will cause the appropriate values in 34^{th} row of the data array T(34,J) and the FOX array (i.e. the coefficients and weights (t=160 seconds)) to be stored in the matrices A, H_1 and Q and in the vectors B_2, B_3, D_1 and D_2 and the scalar $\bar{v}_\omega$. For the forward difference equations in the subroutine COV, the call statement CALL START (1,NS) will cause the appropriate values in the 1st row of the data array T(1,J) (i.e. the coefficients for t=0 seconds) to be stored in the matrices A and H_1 and in the vectors B_2, B_3, D_1, D_2 and K and in the scalars $\bar{v}_\omega$ and f. Appendix B shows how the column index J of the data array T(I,J) is related to the elements of the matrices A and H_1 and the components of the vectors B_1, B_2, B_3, D_1 and D_2.

Subroutine QOEF

The subroutine QOEF is called by the subroutine GAIN. Its function is to update the coefficient matrices and vectors which appear in the backward difference equations. The call statement CALL QOEF (MI,NS) will cause the appropriate values in the MI^{th} row of the data array T(MI,J) and the FOX array (i.e. the difference for the MI^{th} five second time interval) to be <u>subtracted</u> from the current values of the appropriate elements and components of the coefficient matrices and vectors. The differences are subtracted because we are solving backward in time.

Subroutine COEF

The subroutine COEF is called by the subroutine COV. Its function is essentially the same as subroutine QOEF. However since COV solves a forward difference equation, the appropriate differences are <u>added</u> to the current values of the coefficient matrices and vectors.

Subroutine PRB

The subroutine PRB is called by the main program after a call to subroutine COV. The purpose of this subroutine is to provide appropriate calls to the subroutines PAR, END, DRV and REND and to output the results of calculations performed in these subroutines. The subroutines PAR and DRV and subroutines END and REND perform essentially the same calculations. The difference is that the subroutines PAR and END were programmed assuming that the cost functional J^* is a function of response covariances only, whereas the subroutines DRV and REND were programmed under the assumption that the cost functional J^* is a function of response covariances and mean responses. For the current problem only the results of the subroutines PAR and END are used in the optimization iteration procedure (see reference 2 pages 19 and 20). If mean-responses have been calculated in the subroutine COV, then both sets of calculations are performed (i.e. both sets of subroutines are called). (Both results are calculated and printed to show how little effect the mean-responses have on the total cost.) If the calculations for mean responses have been omitted in subroutine COV, then only the subroutines PAR and END are called.

Subroutine DRV

The subroutine DRV is called by the subroutine PRB. The purpose of this subroutine is to compute the part of the cost functional J^* and the derivatives of J^* due to the in-flight constraints on β, I_{B1}, I_{B2}, and I_{B3}.

Cost Functional J^*

Let J^* be the sum

$$J^* = \sum_{i=1}^{m} P(\bar{a}_i) + \sum_{j=m+1}^{n} E\{N_j\} \tag{2.17}$$

Where $P(\bar{a}_i)$ is the likelihood that the i^{th} response exceed its terminal limit γ_i

$$|r_i(T)| > \gamma_i$$

and $E\{N_j\}$ is the expected number of occasions that the j^{th} response exceeds its limit γ_j in the course of the booster flight. Assuming there are m responses whose terminal behavior is to be constrained and n-m responses whose in-flight behavior is to be constrained, it is shown in reference 1 that J^* is an upper bound on the likelihood that one or more responses of interest exceeds its limit:

$$J^* \geq \text{(the likelihood of mission failure)}.$$

Analytical expressions for $P(\bar{a}_i)$ and $E\{N_j\}$ are derived in Appendix E, reference 1 under the assumption that $r_i(T)$ and $r_i(t)$ are Gaussian random variables with a nonzero mean. It is also shown in reference 1 that J^* may be written in the form

$$J^* = f_1(S(T), R(T)) + \int_0^T f_2(S(t), R(t))\, dt \tag{2.18}$$

where the functional f_1 is the sum of the terminal likelihoods $P(\bar{a}_i)$, and the functional f_2 is the sum of the in-flight expectations $E\{N_j\}$. The gimbal and bending expectation densities

$$\frac{\partial}{\partial t} E\{N_\beta\},\ \frac{\partial}{\partial t} E\left\{N_{I_{B1}}\right\},\ \frac{\partial}{\partial t} E\left\{N_{I_{B2}}\right\} \text{ and } \frac{\partial}{\partial t} E\left\{N_{I_{B3}}\right\}$$

for (t=0,5,10,...,155,160 seconds) are calculated and stored in the array P(I,J). Again the index I refers to the 33 time points (t=0,5,...,155,160) and the index J denotes the individual densities.

$$\frac{\partial}{\partial t} E\{N_\beta\} = P(I,1), \qquad \frac{\partial}{\partial t} E\left\{N_{I_{B2}}\right\} = P(I,3)$$

$$\frac{\partial}{\partial t} E\left\{N_{I_{B1}}\right\} = P(I,2), \qquad \frac{\partial}{\partial t} E\left\{N_{I_{B3}}\right\} = P(I,4)$$

The partial derivatives are stored in the array DP(I,J,K) for (t=0,5,10,...,155, 160).

$$\frac{\partial f_2}{\partial S_{11}} = DP(I,1,1) \qquad \frac{\partial f_2}{\partial \bar{r}_1} = DP(I,3,1)$$

$$\frac{\partial f_2}{\partial S_{12}} = DP(I,1,2) \qquad \frac{\partial f_2}{\partial \bar{r}_2} = DP(I,3,2)$$

$$\frac{\partial f_2}{\partial S_{22}} = DP(I,2,2)$$

$$\frac{\partial f_2}{\partial S_{33}} = DP(I,3,3) \qquad \frac{\partial f_2}{\partial \bar{r}_3} = DP(I,1,3)$$

$$\frac{\partial f_2}{\partial S_{34}} = DP(I,3,4)$$

$$\frac{\partial f_2}{\partial \bar{r}_4} = DP(I,1,4)$$

$$\frac{\partial f_2}{\partial S_{44}} = DP(I,4,4)$$

$$\frac{\partial f_2}{\partial S_{55}} = DP(I,5,5) \qquad \frac{\partial f_2}{\partial \bar{r}_5} = DP(I,1,5)$$

$$\frac{\partial f_2}{\partial S_{56}} = DP(I,5,6)$$

$$\frac{\partial f_2}{\partial \bar{r}_6} = DP(I,1,6)$$

$$\frac{\partial f_2}{\partial S_{66}} = DP(I,6,6)$$

$$\frac{\partial f_2}{\partial S_{77}} = DP(I,7,7) \qquad \frac{\partial f_2}{\partial \bar{r}_7} = DP(I,1,7)$$

$$\frac{\partial f_2}{\partial S_{78}} = DP(I,7,8) \qquad \frac{\partial f_2}{\partial \bar{r}_8} = DP(I,1,8)$$

$$\frac{\partial f_2}{\partial S_{88}} = DP(I,8,8)$$

where S_{ij} is i,j^{th} element of the response covariance matrix and $\bar{r}_i$ is the i^{th} component of the mean-response vector.

Subroutine REND

The subroutine REND is called by the subroutine PRB. Its purpose is to calculate the terminal likelihoods $P(\bar{a}_i)$ and the partial derivatives of the terminal likelihoods. The terminal likelihoods are stored in the array CST(I).

$$P(\bar{a}_\alpha) = CST(5) \qquad P(\bar{a}_\phi) = CST(8)$$

$$P(\bar{a}_\phi) = CST(6) \qquad P(\bar{a}_z) = CST(9)$$

$$P(\bar{a}_z) = CST(7)$$

The partial derivatives are stored in the two arrays DCST(I) and RCST(I).

$$\frac{\partial f_1}{\partial S_{99}} = DCST(5) \qquad \frac{\partial f_1}{\partial \bar{r}_9} = RCST(5)$$

$$\frac{\partial f_1}{\partial S_{10,10}} = DCST(6) \qquad \frac{\partial f_1}{\partial \bar{r}_{10}} = RCST(6)$$

$$\frac{\partial f_1}{\partial S_{11,11}} = DCST(7) \qquad \frac{\partial f_1}{\partial \bar{r}_{11}} = RCST(7)$$

$$\frac{\partial f_1}{\partial S_{12,12}} = DCST(8) \qquad \frac{\partial f_1}{\partial \bar{r}_{12}} = RCST(8)$$

$$\frac{\partial f_1}{\partial S_{13,13}} = DCST(9) \qquad \frac{\partial f_1}{\partial \bar{r}_{13}} = RCST(9)$$

Subroutines PAR and END

These subroutines perform the same function as subroutines DRV and REND. The equations for the terminal likelihoods, the in-flight expectations and the derivatives are greatly simplified by assuming that the mean response is zero. Also, of course, the derivatives with respect to $\bar{r}$ are not computed.

SECTION III

INPUT AND OUTPUT

In the first part of the section the input requirements are described for the two types of data decks. In the last part of the section the OUTPUT that can be expected for each of the two data decks is described with the aid of reproduced output listings.

Input

All of the INPUT takes place in the main program. The logical tape number in all of the read statements is the integer variable ICR. ICR is currently set equal to 5 at the beginning of the main program. All units are in the MKS system and all angles are in radians.

As stated in Section II the quadratic theory used in the study requires state equations expressed as first order vector differential equations of the form

$$\dot{x}(t) = F(t)x(t) + G_1(t)u(t) + G_2(t)\bar{v}_\omega(t) + G_3(t)\eta(t)$$

and responses to be controlled expressed as

$$r(t) = H(t)x(t) + D_1(t)u(t) + D_2(t)\bar{v}_\omega(t).$$

Values of the elements of the coefficient matrices $F(t)$ and $H(t)$ and of the components of the coefficient vectors $G_1(t)$, $G_2(t)$, $G_3(t)$, $D_1(t)$ and $D_2(t)$ have been obtained for $t = 0, 5, 10, \ldots, 155$, and 160 seconds. (The method used to obtain the coefficients is given in reference 2, Section II.) The coefficient data is the major portion of the data for the optimization program. It consists of fifteen hundred and eighteen (1518) punched cards. There are six coefficients on each card, in addition to some information to identify the time and the coefficients. All of the coefficient data is read into a "data array" T(I,J) (the dimensions of the "data array" are T(34,292)). The following is a description of a typical data card: The first two columns contain an integer value "I" in the interval $1 \leq I \leq 33$. "I" is used to identify the time associated with the coefficients on a particular card (i.e. $t(I) = 5(I-1)$). In addition, "I" identifies the row number of the "data array" T(I,J) in which the coefficients will be stored. Columns 3, 4, 5 and 6, 7, 8 contain integer values JB and JE in the interval $1 \leq JB, JE \leq 276$. JB and JE assign a "coefficient number" to each of the coefficients on a card in the following manner:

1) The coefficient in columns 9-20 is assigned the number JB

2) The coefficient in columns 21-32 is assigned the number JB + 1

3) " " " " 33-44 " " " " JB + 2

4) " " " " 45-46 " " " " JB + 3

5) " " " " 57-68 " " " " JB + 4

6) " " " " 69-80 " " " " JE.

The "coefficient number" is also the column number J of the data array T(I,J) in which the coefficient will be stored. Since each card has sufficient information to place the coefficients on that card in a particular location within the "data array", the ordering of the coefficient data deck is immaterial. The coefficient data is terminated with a <u>blank card</u>. The FORTRAN format statement used to read in the coefficient data is: 9 FORMAT (I2, 2I3, 6E12.6).

The next card in the data deck contains the value of the integer variable "IGAIN". The value of IGAIN determines the make up of the remainder of the data deck. The FORTRAN format statement used to read in IGAIN is: 9731 FORMAT (I1).

If IGAIN is greater than one (IGAIN > 1), the program will expect to read <u>only two</u> more cards. The first card contains five floating point numbers which will be assigned to the real variables named Q9, Q10, Q11, Q12 and Q13 respectively. These variables are the quadratic weights on the responses which have terminal constraints. The FORTRAN format statement used to read in Q9, Q10, Q11, Q12 and Q13 is: 29 FORMAT (5E12.5). The second card (of the final two cards mentioned above) contains twelve floating point numbers which will be assigned to the real variables named C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11 and C12 respectively. These variables are used in calculating the quadratic weights associated with the responses which have in-flight constraints (see reference 2 Section IV). The FORTRAN format statement used to read in C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11 and C12 is: 30 FORMAT (12F6.3)

If IGAIN is less than or equal to one (IGAIN ≤ 1), the program expects to read the components of the 22 x 1 gain vector K(t) and the scalar deterministic input f(t) for t = 0, 1, 2,..., 158, 159, 160 seconds. The gains and deterministic inputs are read into an array BK(I,J) with dimensions of BK(161,23). For each of the 161 time points, there are <u>four cards</u>. The first three cards contain the first eighteen components of the gain vector. Card one contains components one thru six (1-6), card two contains components seven thru twelve (7-12) and card three contains components thirteen thru eighteen (13-18). The fourth card (in the group representing one time point) contains components nineteen thru twenty two (19-22) plus the "deterministic input". In addition to the floating point numbers representing values for the components of the gains vector and deterministic input, the four cards mentioned above contain integer fields which identify the time point and the component number for the gains vector and deterministic input (the deterministic input is considered to be the twenty-third (23rd) component of a 23 x 1 vector representing the gains vector plus the deterministic input). The FORTRAN format statements for the four cards are: Cards one, two and three - 3003 FORMAT (I3,6(I2,E10.3)) and card four - 3004 FORMAT (I3,5(I2,E10.3)). The total number cards for this part of the data deck is six hundred and forty four (644).

The data deck for the case where IGAIN > 1 contains fifteen hundred and twenty-two (1522) cards as shown in figure 1.

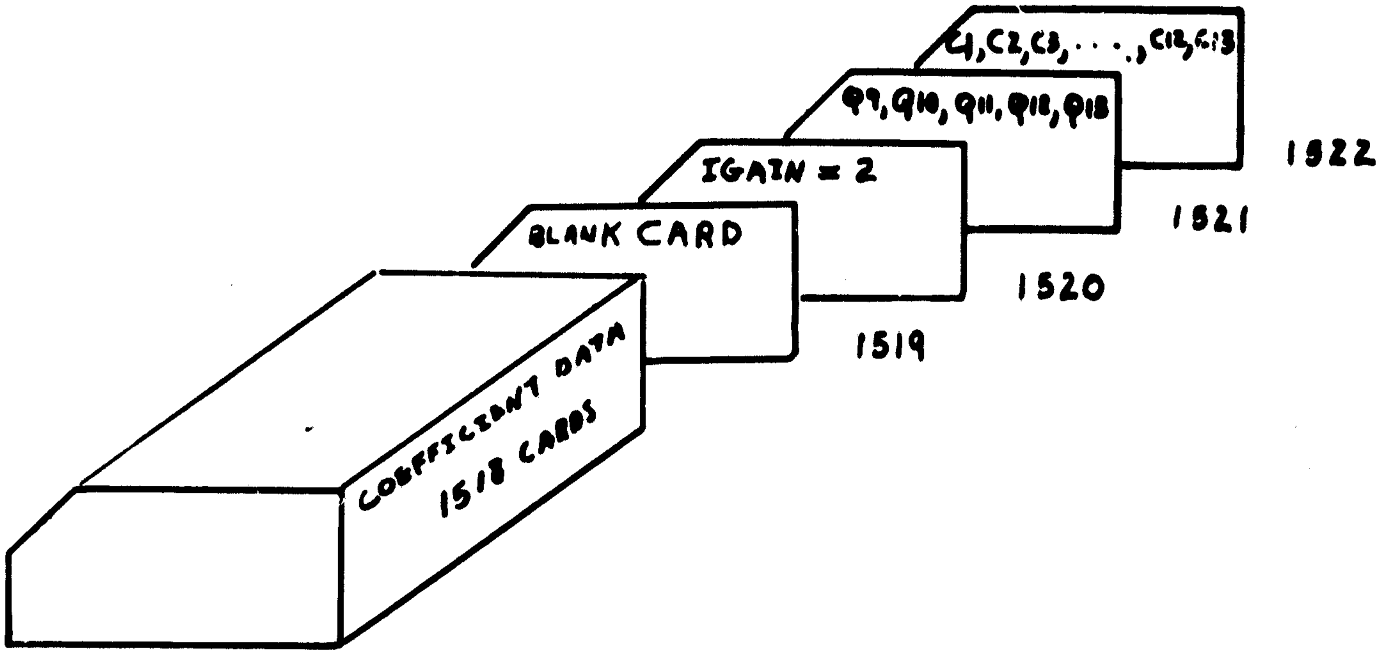

FIGURE 1 DATA DECK FOR IGAIN GREATER THAN ONE

The data deck for the case where IGAIN ≤ 1 contains two thousand, one hundred and sixty-four (2164) cards as shown in figure 2.

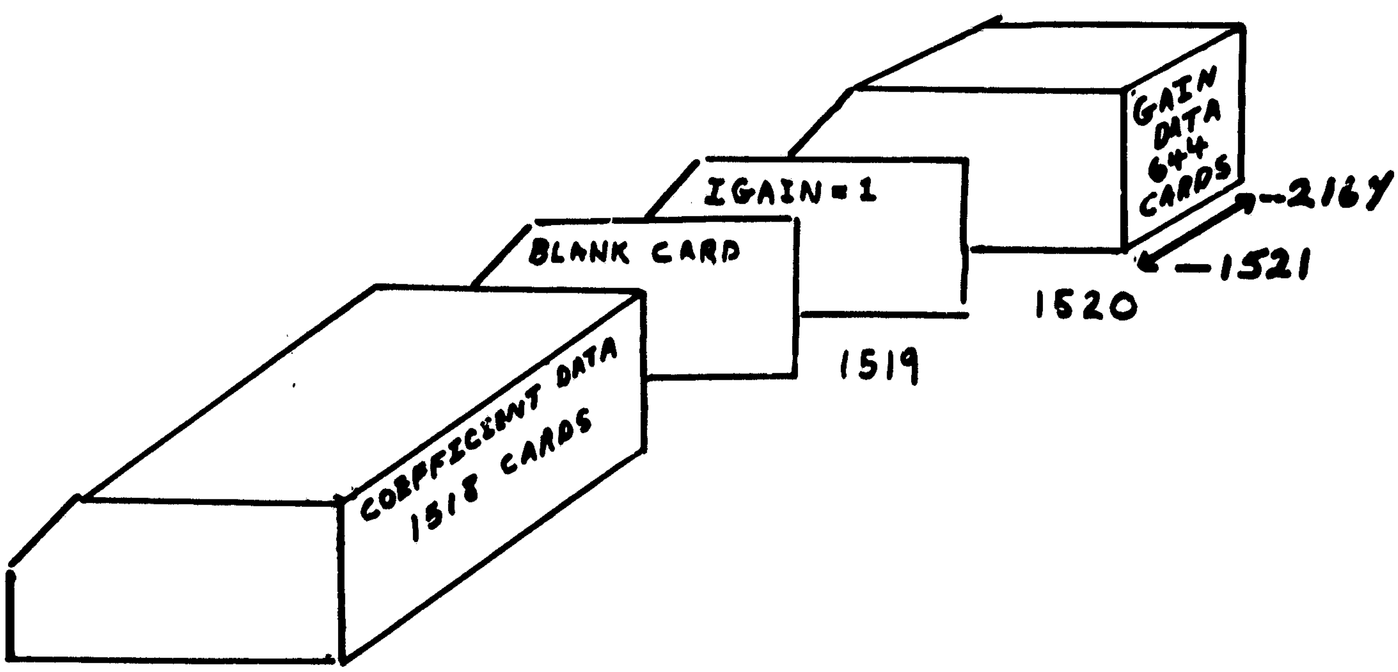

FIGURE 2 DATA DECK FOR IGAIN LESS THAN OR EQUAL TO ONE

Output

Output for the optimization program occurs in the MAIN PROGRAM and in subroutine subprograms GAIN, COV and PRB. The logical tape number in all of the write statements is the integer variable ICP. ICP is currently set equal to 9 at the beginning of the main program.

Main Program

The first output appears in the main program if IGAIN is greater than one ($IGAIN > 1$). The output data is: Q9 thru Q13 (the quadratic weights for responses which have terminal constraints), and the Q array which is composed of the constant and time-varying quadratic weights associated with the in-flight constraints. Figures 3 and 4 show the output as it appears on an actual computer listing. The columns of numbers under the heading Q array in figure 3 and the columns in figure 4 represent the constant and time-varying elements of the matrix $Q(t)$ (which appears in the cost J^{**}) for $t = 0, 5, 10, \ldots, 155, 160$ seconds. Columns one thru six in figure 3 are the elements $Q_{11}(t)$, $Q_{12}(t)$, $Q_{22}(t)$, $Q_{33}(t)$, $Q_{34}(t)$ and $Q_{44}(t)$ which are associated with β, $\dot{\beta}$, I_{B1}, $\dot{I}_{B1}$. Columns one thru six in figure 4 are the elements $Q_{55}(t)$, $Q_{56}(t)$, $Q_{66}(t)$, $Q_{77}(t)$, $Q_{78}(t)$ and $Q_{88}(t)$ which are associated with I_{B2}, $\dot{I}_{B2}$, I_{B3} and $\dot{I}_{B3}$ (note that all off-diagonal elements in the Q array are zero).

Gain Subroutine

In the original difference equation formulation of our problem, the "simple difference approximation" (equation (2.3)) to equation (2.1) was used. For the "simple difference approximation" the quantity FLEX (which is printed out for t = 0,5,10,155 seconds in figure 5) is a measure of the sample approximation error. Initial computer runs indicated that the "simple difference approximation" was not accurate enough. Consequently, the more accurate "sample-data approximation" (equation (2.4)) to equation (2.1) was used for the high frequency components of the state (i.e. flexure modes, gimbal deflection, wind states, and states associated with the load distribution) and the "simple difference approximation" was retained for the low frequency components of the state (rigid body and slosh modes). With the employment of the two types of approximations (the "simple difference approximation" and the "sample-data approximation") the quantity FLEX is no longer a true measure of the sample approximation error and the output shown in figure 5 can be ignored. The computation and output of the FLEX have been left in the program as a check on future models (i.e. to determine if the "simple difference approximation" is sufficiently accurate for future models).

Figure 6 shows the main output for the GAIN subroutine in abreviated form. The 22 x 1 gain vector $K(t)$ and the scalar deterministic input $f(t)$ are printed our for the one hundred and sixty-one (161) time points $t = 0, 1, 2, 3, \ldots, 159, 160$ seconds. The components of the gain vector $K(t)$ are ordered in the same way as the state vector $x(t)$ (see equation (2.1).

$$K(t) =$$

$$[K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{13}, K_{14}, K_{15}, K_{16}, K_{17}, K_{18}, K_{19}, K_{20}, K_{21}, K_{22}] =$$

$$[K_{\dot{\phi}}, K_{\dot{z}}, K_{\dot{\eta}_1}, K_{\dot{\eta}_2}, K_{\dot{\eta}_3}, K_{\dot{z}_{s1}}, K_{\dot{z}_{s2}}, K_{\dot{z}_{s3}}, K_{\phi}, K_z, K_{\eta_1}, K_{\eta_2}, K_{\eta_3}, K_{z_{s1}}, K_{z_{s2}}, K_{z_{s3}}, K_{\beta}, K_{\omega}, K_x, K_{x_1}, K_{x_2}, K_{x_3}] \tag{3.5}$$

Q MATRIX

Q9 THRU Q13

Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}
2.8400E+09	-0.	2.0410E+03	-0.	1.1111E-02

Q ARRAY

Q_{11}	Q_{12}	Q_{22}	Q_{33}	Q_{34}	Q_{44}
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12
7.8799E+05	0.	7.8799E+05	1.2346E-10	0.	7.7160E-12

FIGURE 3 OUTPUT-QUADRATIC WEIGHTS

Q_{55}	Q_{56}	Q_{66}	Q_{77}	Q_{78}	Q_{88}
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0074E-07	0.	1.2523E-10
3.4625E-09	0.	2.1641E-10	4.6696E-07	0.	1.4592E-10
3.5371E-09	0.	2.2107E-10	2.6174E-06	0.	8.1792E-10
4.3968E-09	0.	2.7480E-10	2.7413E-05	0.	8.5665E-09
7.6577E-09	0.	4.7860E-10	1.2146E-04	0.	3.7957E-08
1.0381E-08	0.	6.4879E-10	2.0000E-04	0.	6.2500E-08
7.6577E-09	0.	4.7860E-10	1.2146E-04	0.	3.7957E-08
4.3968E-09	0.	2.7480E-10	2.7413E-05	0.	8.5665E-09
3.5371E-09	0.	2.2107E-10	2.6174E-06	0.	8.1792E-10
3.4625E-09	0.	2.1641E-10	4.6696E-07	0.	1.4592E-10
3.4602E-09	0.	2.1626E-10	4.0074E-07	0.	1.2523E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10
3.4602E-09	0.	2.1626E-10	4.0000E-07	0.	1.2500E-10

FIGURE 4 OUTPUT-QUADRATIC WEIGHTS

```
FLEX= 0.
FLEX= 1.75189277E-01
FLEX= 1.79581766E-01
FLEX= 1.83494190E-01
FLEX= 1.88068340E-01
FLEX= 2.01097490E-01
FLEX= 2.16413956E-01
FLEX= 2.35652443E-01
FLEX= 2.58781431E-01
FLEX= 2.79640266E-01
FLEX= 2.95152003E-01
FLEX= 3.15810254E-01
FLEX= 3.32480106E-01
FLEX= 3.65643270E-01
FLEX= 6.29304562E-01
FLEX= 1.52999567E+00
FLEX= 3.08390007E+00
FLEX= 3.60676385E+00
FLEX= 2.35940737E+00
FLEX= 1.13699034E+00
FLEX= 5.33524496E-01
FLEX= 2.91068333E-01
FLEX= 2.32661740E-01
FLEX= 2.08582060E-01
FLEX= 1.86854671E-01
FLEX= 1.67452664E-01
FLEX= 1.49487764E-01
FLEX= 1.33640732E-01
FLEX= 1.19427956E-01
FLEX= 1.06817710E-01

FLEX= 9.68972147E-02
FLEX= 8.96286378E-02
```

FIGURE 5 OUTPUT MEASURE OF SAMPLE APPROXIMATION ERROR

GAINS AND DETERMINISTIC INPUT

```
   1

K 1= 1.238E-01 K 2= 2.008E-04 K 3=-9.745E-04 K 4= 5.097E-03 K 5=-1.573E-03 K 6=-4.
K 9= 3.226E-02 K10= 7.401E-09 K11= 9.127E-02 K12=-2.184E-01 K13= 3.185E-01 K14=-3.
K17= 2.733E-01 K18=-8.853E-04 K19=-4.754E-01 K20= 1.138E-04 K21=-1.880E-04 K22=-
DET.INPUT=-4.219E-04

   2

K 1= 1.330E-01 K 2= 2.248E-04 K 3=-1.046E-03 K 4= 5.177E-03 K 5=-1.589E-03 K 6=-4.
K 9= 3.572E-02 K10= 7.279E-09 K11= 9.179E-02 K12=-2.176E-01 K13= 3.177E-01 K14=-
K17= 2.682E-01 K18=-8.435E-04 K19=-5.592E-01 K20= 6.378E-05 K21=-2.495E-04 K22=-3.
DET.INPUT=-4.923E-04

   3

K 1= 1.437E-01 K 2= 2.481E-04 K 3=-1.126E-03 K 4= 5.262E-03 K 5=-1.608E-03 K 6=-5
K 9= 3.980E-02 K10= 7.226E-09 K11= 9.229E-02 K12=-2.168E-01 K13= 3.169E-01 K14=-3.
K17= 2.627E-01 K18=-7.859E-04 K19=-6.503E-01 K20=-5.477E-08 K21=-3.004E-04 K22=-2.
DET.INPUT=-5.700E-04

   .
   .
   .
   .
   .
   .

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K 1=-4.357E-04 K 2=-3.758E-09 K 3= 8.502E-04 K 4= 1.446E-03 K 5= 6.488E-03 K 6= 3.
K 9= 9.391E-06 K10= 0.          K11=-6.831E-05 K12=-2.596E-04 K13=-2.412E-03 K14= 9
K17= 9.975E-01 K18=-3.281E-05 K19= 0.          K20= 7.684E-03 K21=-2.261E-03 K22=-3
DET.INPUT= 0.

MAX SAMP APPROX ERROR =  360.766 PERCENT
TIME OF MAX ERROR =   74.990SEC
```

FIGURE 6 OUTPUT GAINS AND DETERMINISTIC INPUT

The final two lines of output in figure 6

MAX SAMP APPROX ERROR = ...

TIME OF ERROR = ...

have meaning only if the simple difference approximation proves to be sufficiently accurate for the problem at hand (in our case this is not true). Consequently these two lines of output can be ignored. In addition to the printed output of the gains vector and the deterministic input, this same data is punched on cards. The format for the punched cards is given under INPUT for the case when IGAIN is less than or equal to one (IGAIN ≤ 1).

COV Subroutine

The output for the covariance subroutine has an option and it depends on the integer variable "II" which appears as an argument in the call to the subroutine. If II is less than or equal to one (II ≤ 1) the first output to appear will be the mean responses (figures 7 and 8) $\bar{r}(t)$ for the thirty-three (33) time points $t = 0, 5, 10, \ldots, 155, 160$ seconds. Following the mean responses are the response covariances (figures 9, 10 and 11) $S(t) = E\{[r(t)-\bar{r}(t)][r(t)-\bar{r}(t)]'\}$ for the same thirty-three time points mentioned previously. If "II" is greater than one ("II" > 1), the mean responses will not be computed and will not appear in the output sequence so that the first output to appear will be the response covariances. In addition to the printed output mentioned above, the covariance subroutine punches cards for the upper triangular part of the (symmetric 22×22) state covariance matrix $X(t)$ for $t = 5, 10, \ldots, 155, 160$ seconds. Since there are $n(n+1)/2$ elements in the upper triangular part of a symmetric matrix of order n, for $n = 22$ we have $22(22+1)/2 = 253$ elements to punch out for each time point. The elements are punched out by row with 8 elements to a card. The FORTRAN format statement used in the conjunction with the punch command is 3050 FORMAT (8E10.3)

PRB Subroutine

The data that is output in subroutine PRB depends on the two integer variables LB and LE which appear as arguments in the call to the subroutine (i.e. CALL PRB (LB,LE).† If mean responses have been computed in the covariance subroutine then the PRB subroutine should be entered with CALL PRB (1,2). If mean responses have not been computed in the covariance subroutine then the PRB subroutine should be entered with CALL PRB (1,1). The output that results from CALL PRB (1,2) is shown in figures 12 thru 21. If CALL PRB (1,1) is used, only the output shown in figures 12 thru 16 will be printed. The data shown in figures 12 thru 16 is generated from calculations performed in subroutines PAR and END. The output shown in figures 17 thru 21 is generated from the calculations performed in subroutines DRV and REND. The difference in the two sets of results is that the cost functional J^*, is considered to be a function of only response covariances in subroutines PAR and END while in subroutines DRV and REND it is considered to be a function of both response covariances and mean responses. The data on the first line in figure 12 is the total cost J^* and the contributions to the total cost due to the in-flight constraints on β, I_{B1}, I_{B2} and I_{B3} and the contributions

† The card in the main program which contains the call statement to the PRB subroutine must be changed manually if the arguments LB and LE are to be changed (i.e. the card reads wither CALL PRB (1,2) or CALL PRB (1,1)).

to the total cost due to the terminal constraints on α, $\dot{\phi}$, $\dot{z}$, ϕ and z. The data in figure 12 under END DERIVATIVES subheading SIG are the partial derivatives $\frac{\partial f_1}{\partial S_{i,i}}$ ($i = 9,10,11,12,13$) where S is the response covariance matrix and f_1 is that part of the cost functional due to the terminal constraints. The data in the same figure under subheading MEAN are the partial derivatives $\frac{\partial f_1}{\partial \bar{r}_i}$ ($i = 9,10,11,12,13$) where $\bar{r}$ is the vector of mean responses. In figures 13, 14, 15 and 16, data is listed for the 33 time points $t = 0, 5, 10, \ldots, 155, 160$. The data in the first columns are the expectation densities

$$\frac{\partial}{\partial t} E\left\{N_\beta\right\}, \frac{\partial}{\partial t} E\left\{N_{I_{B1}}\right\}, \frac{\partial}{\partial t} E\left\{N_{I_{B2}}\right\} \text{ and } \frac{\partial}{\partial t} E\left\{N_{I_{B3}}\right\}.$$

The data in the next five columns for figures 13 thru 16 are the partial derivatives

$$\frac{\partial f_2}{\partial S_{i,i}}, \frac{\partial f_2}{\partial S_{i,(i+1)}}, \frac{\partial f_2}{\partial S_{(i+1),(i+1)}}, \frac{\partial f_2}{\partial \bar{r}_i}, \frac{\partial f_2}{\partial \bar{r}_{(i+1)}}.$$

Where f_2 is that part of the cost functional due to the in-flight constraints and $i = 1$ for 13, $i = 3$ for 14, $i = 5$ for 15 and $i = 7$ for 16. Note that all of the partial derivatives with respect to $\bar{r}$ in figures 12 thru 16 are zero. In figures 17 thru 21 these same partials have nonzero values.

MEAN RESPONSES

	IB1	DIB1	IB2	DIB2	IB3	DIB3
1	0.	0.	0.	0.	0.	0.
2	3.418E+04	8.580E+02	1.517E+04	-3.081E+03	3.457E+03	8.343E+02
3	1.238E+03	-3.459E+03	-3.710E+02	-1.016E+03	1.089E+02	-1.851E+02
4	-3.450E+03	-2.084E+02	-2.114E+03	-2.530E+02	-7.739E+02	-2.939E+02
5	-3.679E+03	-1.088E+03	-2.091E+03	-1.089E+03	-9.117E+02	-5.984E+02
6	-3.241E+03	-2.084E+03	-2.330E+03	-9.547E+02	-1.009E+03	-4.104E+02
7	6.660E+02	-4.113E+03	-2.273E+03	2.391E+02	-1.121E+03	8.311E+02
8	8.638E+03	-9.061E+03	-2.422E+03	2.807E+03	-1.211E+03	3.432E+03
9	2.491E+04	-1.736E+04	-1.710E+03	7.390E+03	-1.420E+03	8.201E+03
10	5.071E+04	-3.001E+04	-2.107E+03	1.572E+04	-1.405E+03	1.647E+04
11	8.897E+04	-4.871E+04	1.522E+03	2.668E+04	-1.852E+03	2.690E+04
12	1.423E+05	-6.524E+04	1.393E+04	4.263E+04	-2.402E+03	4.352E+04
13	1.876E+05	-7.278E+04	-6.144E+03	7.557E+04	-2.405E+03	6.700E+04
14	2.931E+05	-1.029E+05	5.129E+04	9.296E+04	-1.546E+03	7.820E+04
15	4.342E+05	-1.736E+05	1.628E+05	1.036E+05	9.787E+01	1.054E+05
16	3.495E+05	-1.795E+05	8.475E+04	1.004E+05	5.944E+01	1.174E+05
17	3.786E+05	-1.146E+05	1.327E+05	7.501E+04	1.111E+03	1.114E+05
18	3.025E+05	-7.054E+04	1.055E+05	5.747E+04	3.387E+03	9.950E+04
19	1.419E+05	-8.456E+04	5.506E+04	3.681E+03	4.529E+03	4.744E+04
20	-3.156E+04	-2.822E+04	-1.688E+04	-2.658E+04	-3.636E+02	-7.746E+03
21	-1.332E+05	1.632E+03	-6.446E+04	-3.734E+04	-2.054E+03	-4.711E+04
22	-1.281E+05	2.917E+03	-6.045E+04	-3.116E+04	-1.204E+03	-5.538E+04
23	-8.291E+04	1.304E+03	-3.886E+04	-1.600E+04	9.174E+01	-3.816E+04
24	-5.053E+04	-2.107E+03	-2.294E+04	-1.075E+04	-1.871E+02	-2.763E+04
25	-2.953E+04	-1.383E+03	-1.188E+04	-6.886E+03	-3.391E+02	-1.760E+04
26	-1.544E+04	2.917E+02	-4.793E+03	-3.100E+03	-5.856E+02	-8.709E+03
27	-8.493E+03	9.065E+02	-2.025E+03	-2.512E+03	-7.468E+02	-5.226E+03
28	-4.501E+03	2.278E+03	-8.458E+02	-1.837E+03	-8.865E+02	-3.440E+03
29	-3.111E+03	2.598E+03	-1.294E+03	-1.232E+03	-9.817E+02	-2.024E+03
30	-2.602E+03	3.094E+03	-1.847E+03	-1.130E+03	-9.853E+02	-1.817E+03
31	-1.598E+03	3.329E+03	-1.581E+03	-8.380E+02	-6.698E+02	-1.696E+03
32	1.125E+02	2.644E+03	1.173E+03	-1.920E+02	2.586E+02	-1.437E+03
33	1.448E+04	-7.565E+03	1.161E+04	-1.473E+03	4.099E+03	1.045E+04

FIGURE 7 OUTPUT MEAN RESPONSES $\bar{I}_{B1}$, $\dot{\bar{I}}_{B1}$, $\bar{I}_{B2}$, $\dot{\bar{I}}_{B2}$, $\bar{I}_{B3}$, $\dot{\bar{I}}_{B3}$

MEAN RESPONSES

	BETA	DBETA	ALPHA	DPHI	DZ	PHI	Z
1	0.	0.	0.	0.	0.	0.	0.
2	1.021E-03	1.919E-04	2.801E-02	2.191E-03	2.182E-01	1.055E-02	1.895E-01
3	8.798E-05	-9.536E-05	2.798E-03	-9.823E-05	1.045E+00	1.378E-02	3.288E+00
4	-1.854E-05	1.570E-05	-3.414E-03	-4.018E-05	1.888E+00	1.333E-02	1.062E+01
5	-7.584E-06	-9.927E-06	-2.950E-03	-2.006E-05	2.735E+00	1.320E-02	2.217E+01
6	1.708E-05	-1.098E-05	6.026E-08	-4.482E-05	3.599E+00	1.302E-02	3.799E+01
7	1.117E-04	-8.326E-06	4.194E-03	-8.782E-05	4.484E+00	1.269E-02	5.819E+01
8	2.908E-04	-4.641E-06	9.353E-03	-1.459E-04	5.396E+00	1.210E-02	8.287E+01
9	5.957E-04	2.832E-05	1.548E-02	-2.095E-04	6.343E+00	1.121E-02	1.122E+02
10	1.042E-03	4.988E-05	2.195E-02	-2.849E-04	7.349E+00	9.963E-03	1.464E+02
11	1.562E-03	6.412E-05	2.801E-02	-3.517E-04	8.438E+00	8.368E-03	1.858E+02
12	2.142E-03	-1.528E-06	3.333E-02	-4.370E-04	9.637E+00	6.391E-03	2.309E+02
13	2.208E-03	-9.859E-05	3.741E-02	-5.172E-04	1.095E+01	4.010E-03	2.824E+02
14	2.260E-03	-2.602E-05	3.959E-02	-5.695E-04	1.235E+01	1.305E-03	3.405E+02
15	4.297E-03	3.100E-04	4.122E-02	-6.397E-04	1.389E+01	-1.738E-03	4.061E+02
16	6.872E-03	4.260E-04	4.098E-02	-6.548E-04	1.551E+01	-4.987E-03	4.796E+02
17	9.429E-03	4.358E-04	4.070E-02	-6.491E-04	1.706E+01	-8.259E-03	5.611E+02
18	8.463E-03	-1.113E-04	3.334E-02	-5.343E-04	1.820E+01	-1.123E-02	6.495E+02
19	3.639E-03	-7.660E-04	1.407E-02	-2.174E-04	1.826E+01	-1.312E-02	7.412E+02
20	-9.983E-04	-8.930E-04	-3.689E-03	6.102E-05	1.702E+01	-1.351E-02	8.299E+02
21	-3.873E-03	-4.095E-04	-1.813E-02	2.988E-04	1.487E+01	-1.256E-02	9.099E+02
22	-3.695E-03	7.578E-05	-2.118E-02	3.541E-04	1.247E+01	-1.087E-02	9.782E+02
23	-2.409E-03	2.545E-04	-1.736E-02	2.975E-04	1.038E+01	-9.222E-03	1.035E+03
24	-1.515E-03	1.521E-04	-1.433E-02	2.546E-04	8.667E+00	-7.847E-03	1.083E+03
25	-9.209E-04	1.044E-04	-1.198E-02	2.138E-04	7.256E+00	-6.678E-03	1.122E+03
26	-5.237E-04	6.887E-05	-1.002E-02	1.858E-04	6.081E+00	-5.682E-03	1.156E+03
27	-2.923E-04	4.645E-05	-8.418E-03	1.505E-04	5.088E+00	-4.841E-03	1.183E+03
28	-1.494E-04	1.787E-05	-6.880E-03	1.125E-04	4.230E+00	-4.184E-03	1.207E+03
29	-6.975E-05	6.798E-06	-5.757E-03	6.055E-05	3.404E+00	-3.751E-03	1.226E+03
30	-3.298E-05	-2.512E-07	-5.005E-03	1.670E-05	2.730E+00	-3.564E-03	1.241E+03
31	-1.621E-05	-3.602E-06	-4.528E-03	-1.277E-05	1.974E+00	-3.582E-03	1.253E+03
32	-2.642E-05	-3.372E-06	-3.992E-03	1.108E-04	1.165E+00	-3.483E-03	1.261E+03
33	4.255E-04	5.312E-06	-4.119E-04	1.362E-03	5.780E-01	-1.775E-04	1.265E+03

FIGURE 8 OUTPUT MEAN RESPONSES $\bar{\beta}$, $\bar{\dot{\beta}}$, $\bar{\alpha}$, $\bar{\dot{\phi}}$, $\bar{\dot{z}}$, $\bar{\phi}$, $\bar{z}$

RESPONSE COVARIANCES

	IB1	IB1*DIB1	DIB1	IB2	IB2*DIB2	DIB2	IB3	IB3*DIB3	DIB3
1	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	6.277E+09	-2.038E+09	6.830E+08	8.433E+08	-6.359E+07	9.033E+06	2.863E+08	1.254E+08	5.555E+07
3	5.491E+08	1.236E+08	2.013E+08	3.867E+08	-8.023E+07	6.341E+07	1.043E+07	-3.234E+06	1.177E+07
4	4.460E+08	-2.931E+07	3.520E+08	1.803E+08	1.588E+07	1.061E+08	2.563E+07	1.083E+07	2.395E+07
5	9.242E+08	-1.554E+08	4.070E+08	2.829E+08	3.199E+07	1.729E+08	5.582E+07	2.116E+07	5.291E+07
6	1.545E+09	-3.696E+08	1.280E+09	3.473E+08	6.498E+07	3.950E+08	9.144E+07	4.113E+07	1.294E+08
7	2.209E+09	-6.883E+08	5.456E+09	3.610E+08	1.050E+08	1.291E+09	1.056E+08	6.152E+07	3.452E+08
8	3.011E+09	-1.072E+09	1.814E+10	3.531E+08	1.275E+08	4.103E+09	1.083E+08	8.095E+07	9.823E+08
9	4.041E+09	-1.588E+09	4.421E+10	4.045E+08	1.3[illegible]E+08	1.010E+10	1.111E+08	9.040E+07	2.434E+09
10	6.293E+09	-2.479E+09	1.102E+11	6.600E+08	1.46[illegible]E+08	2.635E+10	1.598E+08	1.076E+08	6.211E+09
11	1.050E+10	-4.138E+09	2.445E+11	1.238E+09	1.382E+08	5.837E+10	2.799E+08	1.068E+08	1.448E+10
12	1.854E+10	-7.550E+09	5.241E+11	2.869E+09	9.352E+07	1.472E+11	7.477E+08	6.601E+07	4.401E+10
13	2.513E+10	-1.181E+10	9.708E+11	5.391E+09	-1.468E+09	4.133E+11	1.425E+09	-2.924E+08	1.184E+11
14	4.389E+10	-1.824E+10	1.841E+12	6.633E+09	-4.643E+08	5.244E+11	2.067E+09	-7.286E+08	2.222E+11
15	9.761E+10	-3.495E+10	5.069E+12	1.661E+10	2.802E+09	9.894E+11	4.919E+09	-1.744E+09	5.381E+11
16	1.528E+11	-5.932E+10	9.795E+12	2.516E+10	-4.514E+09	1.817E+12	7.058E+09	-3.056E+09	7.362E+11
17	1.440E+11	-6.058E+10	8.510E+12	2.422E+10	-4.491E+09	1.226E+12	6.160E+09	-2.864E+09	5.751E+11
18	1.558E+11	-5.260E+10	8.741E+12	2.896E+10	-5.639E+09	1.626E+12	7.528E+09	-1.653E+09	6.171E+11
19	2.069E+11	-5.747E+10	1.160E+13	4.269E+10	-3.762E+09	2.529E+12	1.286E+10	1.972E+09	8.804E+11
20	4.758E+10	-2.427E+10	2.480E+12	1.008E+10	-3.487E+09	5.803E+11	3.507E+09	-1.039E+09	2.521E+11
21	1.009E+10	-3.190E+09	3.768E+11	2.149E+09	-1.130E+08	1.002E+11	5.186E+08	-1.520E+08	4.477E+10
22	3.744E+09	-1.071E+09	1.292E+11	7.993E+08	-5.730E+06	3.911E+10	1.737E+08	-6.184E+07	1.449E+10
23	1.398E+09	-2.958E+08	4.347E+10	3.006E+08	1.010E+07	1.566E+10	6.183E+07	-2.876E+07	4.478E+09
24	8.220E+08	-8.551E+07	2.507E+10	1.782E+08	1.949E+07	8.105E+09	4.162E+07	-1.007E+07	2.243E+09
25	4.031E+08	-3.716E+07	1.246E+10	9.220E+07	3.464E+06	2.885E+09	3.240E+07	-6.192E+06	9.149E+08
26	2.754E+08	3.210E+05	5.981E+09	1.020E+08	3.254E+06	1.217E+09	3.669E+07	-1.834E+05	3.139E+08
27	3.017E+08	1.065E+07	2.653E+09	1.746E+08	8.919E+06	3.992E+08	5.000E+07	2.269E+06	1.118E+08
28	4.703E+08	1.628E+07	1.232E+09	3.738E+08	3.355E+07	1.739E+08	7.181E+07	7.180E+06	4.651E+07
29	6.364E+08	-2.780E+06	5.773E+08	7.297E+08	7.547E+07	1.050E+08	9.988E+07	1.293E+07	2.617E+07
30	7.747E+08	-6.945E+07	3.537E+08	1.109E+09	1.014E+08	7.847E+07	1.191E+08	2.167E+07	3.248E+07
31	4.969E+08	-1.591E+08	3.343E+08	1.019E+09	5.278E+07	6.837E+07	1.016E+08	3.104E+07	5.921E+07
32	1.571E+08	-1.528E+08	3.163E+08	2.183E+08	-2.370E+07	5.857E+07	2.134E+07	2.553E+07	7.781E+07
33	4.411E+09	-1.929E+09	3.165E+09	3.088E+09	-4.679E+08	6.969E+08	3.915E+08	8.484E+08	2.168E+09

FIGURE 9 OUTPUT RESPONSE COVARIANCES

$$\left[\sigma_{I_{B1}}^2,\ \rho_{I_{B1},\dot{I}_{B1}}\sigma_{I_{B1}}\sigma_{\dot{I}_{B1}},\ \sigma_{\dot{I}_{B1}}^2,\ \sigma_{I_{B2}}^2,\ \rho_{I_{B2},\dot{I}_{B2}}\sigma_{I_{B2}}\sigma_{\dot{I}_{B2}},\ \sigma_{\dot{I}_{B2}}^2,\ \sigma_{I_{B3}}^2,\ \rho_{I_{B3},\dot{I}_{B3}}\sigma_{I_{B3}}\sigma_{\dot{I}_{B3}}\ \sigma_{\dot{I}_{B3}}^2\right.$$

RESPONSE COVARIANCES

	BETA	BETA*DBETA	DBETA	ALPHA	DPHI	DZ	PHI	Z
1	0.	0.	0.	0.	0.	0.	0.	0.
2	7.126E-06	-1.433E-06	1.984E-03	1.296E-01	5.902E-07	1.274E-01	1.678E-04	1.250E-01
3	1.236E-07	-7.047E-08	3.359E-05	6.627E-03	1.506E-07	1.293E+00	1.160E-04	1.765E+01
4	6.613E-08	-3.210E-09	1.886E-05	2.950E-03	7.715E-07	3.164E+00	6.195E-05	1.337E+02
5	8.274E-08	4.235E-09	2.364E-05	1.677E-03	3.191E-07	4.885E+00	2.310E-05	4.662E+02
6	1.638E-07	1.364E-08	4.693E-05	1.099E-03	1.452E-07	6.253E+00	7.896E-06	1.115E+03
7	3.245E-07	1.582E-08	9.341E-05	7.621E-04	5.782E-08	7.266E+00	2.277E-06	2.150E+03
8	5.196E-07	2.691E-09	1.508E-04	5.537E-04	3.473E-08	7.983E+00	5.273E-07	3.615E+03
9	7.515E-07	-1.050E-08	2.204E-04	4.477E-04	3.009E-08	8.438E+00	7.011E-07	5.531E+03
10	1.158E-06	-3.935E-08	3.440E-04	4.122E-04	3.785E-08	8.708E+00	2.291E-06	7.902E+03
11	1.693E-06	-1.034E-07	5.109E-04	4.148E-04	4.596E-08	8.855E+00	5.335E-06	1.072E+04
12	2.594E-06	-3.924E-07	8.264E-04	4.244E-04	6.676E-08	8.924E+00	1.010E-05	1.398E+04
13	2.841E-06	-2.421E-06	1.268E-03	4.383E-04	9.462E-08	8.916E+00	1.733E-05	1.765E+04
14	3.420E-06	-9.610E-06	2.805E-03	4.622E-04	1.118E-07	8.861E+00	2.707E-05	2.169E+04
15	1.016E-05	-2.591E-05	7.951E-03	4.328E-04	1.381E-07	8.814E+00	3.888E-05	2.603E+04
16	2.235E-05	-3.575E-05	1.339E-02	5.205E-04	1.613E-07	9.088E+00	5.313E-05	3.058E+04
17	2.824E-05	-2.482E-05	1.287E-02	4.457E-04	1.322E-07	9.569E+00	6.804E-05	3.525E+04
18	2.425E-05	-1.056E-05	8.795E-03	3.087E-04	1.018E-07	9.543E+00	7.944E-05	3.983E+04
19	1.639E-05	-4.029E-06	5.211E-03	1.688E-04	9.561E-08	8.905E+00	8.461E-05	4.404E+04
20	7.463E-06	-1.345E-06	2.232E-03	1.072E-04	5.459E-08	8.826E+00	8.408E-05	4.760E+04
21	4.038E-06	-4.457E-07	1.177E-03	8.548E-05	2.938E-08	1.072E+01	8.045E-05	5.045E+04
22	1.770E-06	-2.355E-07	5.108E-04	5.523E-05	1.974E-08	1.515E+01	7.549E-05	5.278E+04
23	7.940E-07	-9.627E-08	2.279E-04	3.851E-05	1.507E-08	2.227E+01	6.978E-05	5.501E+04
24	4.837E-07	-3.884E-08	1.391E-04	3.949E-05	1.683E-08	3.210E+01	6.391E-05	5.777E+04
25	2.294E-07	-2.696E-08	6.577E-05	3.401E-05	1.842E-08	4.448E+01	5.765E-05	6.187E+04
26	1.021E-07	-1.283E-08	2.928E-05	3.000E-05	2.601E-08	5.911E+01	5.005E-05	6.832E+04
27	4.152E-08	-5.791E-09	1.193E-05	2.450E-05	4.568E-08	7.533E+01	3.995E-05	7.823E+04
28	1.665E-08	-2.489E-09	4.798E-06	2.195E-05	1.066E-07	9.165E+01	2.623E-05	9.277E+04
29	6.312E-09	-1.023E-09	1.834E-06	2.680E-05	2.851E-07	1.048E+02	1.013E-05	1.129E+05
30	2.761E-09	-4.425E-10	8.153E-07	5.076E-05	6.843E-07	1.089E+02	2.142E-06	1.390E+05
31	1.630E-09	-1.861E-10	4.896E-07	1.155E-04	1.306E-06	9.587E+01	3.154E-05	1.695E+05
32	1.666E-08	2.180E-09	4.794E-06	2.135E-04	5.285E-07	6.219E+01	1.186E-04	2.005E+05
33	3.580E-06	3.702E-08	1.015E-03	5.880E-06	4.480E-05	3.267E+01	6.719E-06	2.258E+05

FIGURE 10 OUTPUT RESPONSE COVARIANCES

$|\sigma_\beta|^2$, $|\rho_{\beta,\dot\beta}\sigma_\beta\sigma_{\dot\beta}|$, $\sigma_{\dot\beta}{}^2$, $\sigma_\alpha{}^2$, $\sigma_{\dot\phi}{}^2$ $\sigma_{\dot z}{}^2$, $\sigma_\phi{}^2$, $\sigma_z{}^2$

RESPONSE COVARIANCES

	OMEGA	X	X1	X2	X3
1	0.	0.	0.	0.	0.
2	9.980E-01	9.646E-09	8.629E-02	4.845E-02	4.866E-02
3	9.977E-01	9.644E-09	2.215E-02	9.628E-02	8.657E-02
4	9.975E-01	9.643E-09	9.345E-03	1.114E-02	2.160E-02
5	9.974E-01	9.642E-09	6.076E-03	6.882E-03	1.403E-02
6	9.973E-01	9.641E-09	4.107E-03	4.488E-03	9.408E-03
7	9.973E-01	9.639E-09	2.798E-03	3.004E-03	6.385E-03
8	9.973E-01	9.638E-09	1.936E-03	2.052E-03	4.408E-03
9	9.974E-01	9.638E-09	1.428E-03	1.487E-03	3.239E-03
10	9.976E-01	9.637E-09	1.167E-03	1.197E-03	2.638E-03
11	9.978E-01	9.636E-09	1.047E-03	1.061E-03	2.359E-03
12	9.980E-01	9.637E-09	9.841E-04	9.919E-04	2.211E-03
13	9.983E-01	9.638E-09	9.595E-04	9.659E-04	2.157E-03
14	9.985E-01	9.639E-09	9.724E-04	9.767E-04	2.185E-03
15	9.988E-01	9.640E-09	9.197E-04	9.252E-04	2.068E-03
16	9.990E-01	9.642E-09	1.062E-03	1.061E-03	2.383E-03
17	9.992E-01	9.644E-09	9.437E-04	9.508E-04	2.123E-03
18	9.993E-01	9.645E-09	6.784E-04	6.882E-04	1.529E-03
19	9.995E-01	9.647E-09	3.696E-04	3.775E-04	8.344E-04
20	9.996E-01	9.649E-09	2.033E-04	2.069E-04	4.585E-04
21	9.996E-01	9.649E-09	1.386E-04	1.401E-04	3.121E-04
22	9.996E-01	9.650E-09	7.443E-05	7.553E-05	1.677E-04
23	9.997E-01	9.651E-09	4.368E-05	4.415E-05	9.836E-05
24	9.999E-01	9.652E-09	4.643E-05	4.653E-05	1.044E-04
25	9.999E-01	9.653E-09	4.012E-05	4.027E-05	9.022E-05
26	9.999E-01	9.653E-09	3.599E-05	3.610E-05	8.090E-05
27	9.999E-01	9.654E-09	2.946E-05	2.961E-05	6.631E-05
28	1.000E+00	9.653E-09	2.421E-05	2.433E-05	5.448E-05
29	1.000E+00	9.652E-09	1.998E-05	2.007E-05	4.498E-05
30	1.000E+00	9.653E-09	1.658E-05	1.666E-05	3.734E-05
31	1.000E+00	9.652E-09	1.369E-05	1.373E-05	3.081E-05
32	1.000E+00	9.650E-09	1.139E-05	1.143E-05	2.563E-05
33	1.000E+00	9.649E-09	9.794E-06	9.827E-06	2.204E-05

FIGURE 11 OUTPUT RESPONSE COVARIANCES $[\sigma_\omega]^2$, $[\sigma_x]^2$, $[\sigma_{x_1}]^2$, $[\sigma_{x_2}]^2$, $[\sigma_{x_3}]^2$

```
COSTS
  TOTAL        BETA      IB1        IB2         IB3       ALPHA       DPHI        DZ      PHI        Z
8.7461E-04 0.            0.      2.1691E-14 8.7461E-04 0.            0.       3.8828E-35 0.          0.

END DERIVATIVES
   SIG         MEAN
0.            0.
0.            0.
4.0769E-34    0.
0.            0.
0.            0.
```

FIGURE 12 OUTPUT J^* , $[E\{N_i\}\ (i=1,3,5,7)]$, $[P(\bar{a}_j)\ (j=9,10,\ldots,13)]$, $\frac{\partial f_1}{\partial S_{ii}}\ (i=9,10,\ldots,13)$ - (MEAN RESPONSES NOT INCLUDED)

BETA COST	P	BETA	BETA*DBETA	DBETA	RBETA	RDBETA
1	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.	0.
15	0.	0.	0.	0.	0.	0.
16	0.	0.	0.	0.	0.	0.
17	0.	0.	0.	0.	0.	0.
18	0.	0.	0.	0.	0.	0.
19	0.	0.	0.	0.	0.	0.
20	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.
27	0.	0.	0.	0.	0.	0.
28	0.	0.	0.	0.	0.	0.
29	0.	0.	0.	0.	0.	0.
30	0.	0.	0.	0.	0.	0.
31	0.	0.	0.	0.	0.	0.
32	0.	0.	0.	0.	0.	0.
33	0.	0.	0.	0.	0.	0.

FIGURE 13 OUTPUT - $\frac{\partial E\{N_\beta\}}{\partial t}, \frac{\partial f_2}{\partial S_{11}}, \frac{\partial f_2}{\partial S_{12}}, \frac{\partial f_2}{\partial S_{22}}$

FOR t=0,5,..., 160 SEC (MEAN RESPONSES NOT INCLUDED)

BENDING COST IB1

	P	IB	IB*DIB	DIB	RIB	RDIB
1	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.	0.
15	0.	0.	0.	0.	0.	0.
16	0.	0.	0.	0.	0.	0.
17	0.	0.	0.	0.	0.	0.
18	0.	0.	0.	0.	0.	0.
19	0.	0.	0.	0.	0.	0.
20	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.
27	0.	0.	0.	0.	0.	0.
28	0.	0.	0.	0.	0.	0.
29	0.	0.	0.	0.	0.	0.
30	0.	0.	0.	0.	0.	0.
31	0.	0.	0.	0.	0.	0.
32	0.	0.	0.	0.	0.	0.
33	0.	0.	0.	0.	0.	0.

FIGURE 14 OUTPUT - $\frac{\partial E\left\{N_{I_{B1}}\right\}}{\partial t}$, $\frac{\partial f_2}{\partial S_{33}}$, $\frac{\partial f_2}{\partial S_{34}}$, $\frac{\partial f_2}{\partial S_{44}}$

FOR t=0,5,10,...,155,160 SEC (MEAN RESPONSES NOT INCLUDED)

BENDING COST IB2

	P	IB	IB*DIB	DIB	RIB	RDIB
1	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.	0.
15	5.640E-38	2.927E-46	6.358E-48	1.950E-50	0.	0.
16	2.265E-25	5.155E-34	1.578E-35	8.192E-38	0.	0.
17	1.879E-26	4.623E-35	1.700E-36	1.078E-38	0.	0.
18	3.584E-22	6.161E-31	2.329E-32	1.506E-34	0.	0.
19	4.338E-15	3.401E-24	1.423E-25	9.636E-28	0.	0.
20	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.
27	0.	0.	0.	0.	0.	0.
28	0.	0.	0.	0.	0.	0.
29	0.	0.	0.	0.	0.	0.
30	0.	0.	0.	0.	0.	0.
31	0.	0.	0.	0.	0.	0.
32	0.	0.	0.	0.	0.	0.
33	0.	0.	0.	0.	0.	0.

FIGURE 15 OUTPUT - $\frac{\partial E\left\{N_{I_{B2}}\right\}}{\partial t}$, $\frac{\partial f_2}{\partial S_{5,5}}$, $\frac{\partial f_2}{\partial S_{5,6}}$, $\frac{\partial f_2}{\partial S_{6,6}}$

FOR t=0,5,10,...,155,160 SEC (MEAN RESPONSES NOT INCLUDED)

BENDING COST IB3

	P	IB	IB*DIB	DIB	RIB	RDIB
1	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.
13	1.552E-38	9.545E-46	2.269E-47	9.351E-50	0.	0.
14	1.073E-26	3.142E-34	8.173E-36	3.754E-38	0.	0.
15	2.221E-11	1.140E-19	4.319E-21	2.763E-23	0.	0.
16	4.725E-08	1.176E-16	5.516E-18	4.354E-20	0.	0.
17	3.129E-09	1.028E-17	4.854E-19	3.929E-21	0.	0.
18	1.477E-07	3.197E-16	1.674E-17	1.421E-19	0.	0.
19	1.747E-04	1.240E-13	8.715E-15	8.947E-17	0.	0.
20	6.009E-16	6.095E-24	2.450E-25	1.696E-27	0.	0.
21	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.
27	0.	0.	0.	0.	0.	0.
28	0.	0.	0.	0.	0.	0.
29	0.	0.	0.	0.	0.	0.
30	0.	0.	0.	0.	0.	0.
31	0.	0.	0.	0.	0.	0.
32	0.	0.	0.	0.	0.	0.
33	0.	0.	0.	0.	0.	0.

FIGURE 16 OUTPUT - $\frac{\partial E\left\{N_{I_{B3}}\right\}}{\partial t}, \frac{\partial f_2}{\partial S_{7,7}}, \frac{\partial f_2}{\partial S_{7,8}}, \frac{\partial f_2}{\partial S_{8,8}}$

FOR t=0,5,10,...,155,160 SEC (MEAN RESPONSES NOT INCLUDED)

```
COSTS
  TOTAL       BETA        IB1          IB2          IB3        ALPHA        DPHI           DZ        PHI          Z
8.9783E-04 1.8699E-46 0.          9.5563E-14 8.9783E-04 0.            0.          3.3284E-34 0.           0.

END DERIVATIVES
   SIG        MEAN
0.           0.
0.           0.
-7.5317E-34 5.9831E-34
0.           0.
0.           0.
```

FIGURE 17 OUTPUT - J^*, $[E\{N_i\}\ (i=1,3,5,7)]$, $[P(\bar{a}_j)\ (j=9,10,11,12,13)]$, $\frac{\partial f_1}{\partial S_{i1}}, \frac{\partial f_1}{\partial \bar{r}_i}\ (i=9,10,11,12,13)$

BETA COST

	P	BETA	BETA*DBETA	DBETA	RBETA	RDBETA
1	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.	0.
15	0.	0.	0.	0.	0.	0.
16	0.	0.	0.	0.	0.	0.
17	3.740E-47	3.613E-40	1.484E-42	2.876E-45	-2.450E-44	5.365E-46
18	0.	0.	0.	0.	0.	0.
19	0.	0.	0.	0.	0.	0.
20	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.
27	0.	0.	0.	0.	0.	0.
28	0.	0.	0.	0.	0.	0.
29	0.	0.	0.	0.	0.	0.
30	0.	0.	0.	0.	0.	0.
31	0.	0.	0.	0.	0.	0.
32	0.	0.	0.	0.	0.	0.
33	0.	0.	0.	0.	0.	0.

FIGURE 18 OUTPUT - $\frac{\partial E\{N_\beta\}}{\partial t}, \frac{\partial f_2}{\partial S_{11}}, \frac{\partial f_2}{\partial S_{12}}, \frac{\partial f_2}{\partial S_{22}}, \frac{\partial f_2}{\partial \bar{r}_1}, \frac{\partial f_2}{\partial \bar{r}_2}$

FOR t=0,5,10,...,155,160 SEC

BENDING COST IB1

	P	IB	IB*DIB	DIB	RIB	RDIB
1	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.	0.
15	0.	0.	0.	0.	0.	0.
16	0.	0.	0.	0.	0.	0.
17	0.	0.	0.	0.	0.	0.
18	0.	0.	0.	0.	0.	0.
19	0.	0.	0.	0.	0.	0.
20	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.
27	0.	0.	0.	0.	0.	0.
28	0.	0.	0.	0.	0.	0.
29	0.	0.	0.	0.	0.	0.
30	0.	0.	0.	0.	0.	0.
31	0.	0.	0.	0.	0.	0.
32	0.	0.	0.	0.	0.	0.
33	0.	0.	0.	0.	0.	0.

FIGURE 19 OUTPUT - $\frac{\partial E\left\{N_{I_{B1}}\right\}}{dt}, \frac{\partial f_2}{\partial S_{3,3}}, \frac{\partial f_2}{\partial S_{3,4}}, \frac{\partial f_2}{\partial S_{4,4}}, \frac{\partial f_2}{\partial \bar{r}_3}, \frac{\partial f_2}{\partial \bar{r}_4}$

FOR t=0,5,10,...,155,160 SEC

BENDING COST IB2

	P	IB	IB*DIB	DIB	RIB	RDIB
1	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.	0.
15	2.372E-31	2.489E-39	2.331E-41	7.364E-44	-4.691E-36	2.521E-37
16	3.389E-23	1.542E-31	2.160E-33	1.108E-35	-2.223E-28	3.357E-29
17	8.224E-23	4.056E-31	6.593E-33	4.250E-35	-8.824E-28	1.016E-28
18	7.895E-20	2.714E-28	4.684E-30	3.089E-32	-5.589E-25	8.484E-26
19	1.911E-14	2.977E-23	6.055E-25	4.218E-27	-5.689E-20	1.530E-20
20	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.
27	0.	0.	0.	0.	0.	0.
28	0.	0.	0.	0.	0.	0.
29	0.	0.	0.	0.	0.	0.
30	0.	0.	0.	0.	0.	0.
31	0.	0.	0.	0.	0.	0.
32	0.	0.	0.	0.	0.	0.
33	0.	0.	0.	0.	0.	0.

FIGURE 20 OUTPUT - $\frac{\partial E\{N_{I_{B2}}\}}{\partial t}$; $\frac{\partial f_2}{\partial s_{5,5}}$, $\frac{\partial f_2}{\partial s_{5,6}}$, $\frac{\partial f_2}{\partial s_{6,6}}$, $\frac{\partial f_2}{\partial \overline{r}_5}$, $\frac{\partial f_2}{\partial \overline{r}_6}$

FOR t=0,5,10,...,155,160 SEC

BENDING COST IB3

	P	IB	IB*DIB	DIB	RIB	RDIB
1	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.
13	1.783E-38	1.355E-45	2.718E-47	1.214E-49	-4.706E-42	-4.437E-44
14	1.072E-26	4.483E-34	8.249E-36	3.883E-38	-1.455E-30	-6.739E-33
15	2.256E-11	1.848E-19	4.335E-21	2.738E-23	-9.069E-16	6.230E-18
16	4.789E-08	1.889E-16	5.531E-18	4.316E-20	-1.363E-12	1.052E-14
17	3.251E-09	1.745E-17	4.951E-19	3.922E-21	-9.352E-14	1.417E-15
18	1.585E-07	5.779E-16	1.756E-17	1.451E-19	-3.387E-12	8.611E-14
19	1.794E-04	2.074E-13	8.884E-15	9.061E-17	-2.788E-09	4.825E-11
20	6.025E-16	8.901E-24	2.456E-25	1.699E-27	-4.608E-20	-1.154E-22
21	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.
27	0.	0.	0.	0.	0.	0.
28	0.	0.	0.	0.	0.	0.
29	0.	0.	0.	0.	0.	0.
30	0.	0.	0.	0.	0.	0.
31	0.	0.	0.	0.	0.	0.
32	0.	0.	0.	0.	0.	0.
33	0.	0.	0.	0.	0.	0.

FIGURE 21 OUTPUT - $\frac{\partial E\{N_{I_{B3}}\}}{\partial t}, \frac{\partial f_2}{\partial S_{7,7}}, \frac{\partial f_2}{\partial S_{7,8}}, \frac{\partial f_2}{\partial S_{8,8}}, \frac{\partial f_2}{\partial \bar{r}_7}, \frac{\partial f_2}{\partial \bar{r}_8}$

FOR t=0,5,10,...,155,160 SEC

SECTION IV

RUNNING PROCEDURE

In the first part of this section, the two basic types of runs that can be made with the optimization program are discussed. In the second part of the section, some comments are made on the future use of the optimization program in terms of program memory requirements and running time.

Two Types of Runs

There are two types of runs that can be made with the optimization program. The first is an "optimization iteration" (IGAIN > 1). The second type of run will be referred to as a "simplification run". (IGAIN ≤ 1)

Optimization Iteration

The "optimization iteration" run represents one iteration in the "iteration procedure" for minimizing the upper bound J^* as described in references 1 and 2. An optimization iteration involves:

(a) Choosing the quadratic weights Q - main program

(b) Calculating the quadratic-optimal controller - GAIN SUBROUTINE

(c) Calculating response means and covariances - COV SUBROUTINE

(d) Calculating J^* and partial derivatives $\frac{\partial f}{\partial R}$, $\frac{\partial f}{\partial S}$ - PRB SUBROUTINE

(e) Rechoose quadratic weights Q and repeat (b), (c) and (d).

The above procedure is based on an assertion of quadratic equivalence (reference 1). The assertion states that the linear controller minimizing J^* also minimizes a quadratic functional of the first and second moments (i.e. means and covariances). The coefficients of that quadratic functional are the partial derivatives of J^* with respect to the response moments evaluated with the moments produced by the optimum controller. The above process is repeated until the initially chosen weights and the derived partial derivatives are approximately equal.

Simplification Run

For a simplification run it is assumed that an optimal controller has already been found. The gains for the controller are read into the array BK as described in Section III - INPUT. The gains are differenced and stored in BK as described in Section II - GAIN SUBROUTINE. The simplification run is used to determine degradation in performance caused by deleting the feedback gains of certain states from the optimal controller. This simplified controller is obtained by setting certain components in the gain vector to zero (i.e. certain columns in the array BK(I,J) are set equal to zero). The degradation in performance is determined by comparing the response means and covariances and the value of the cost functional

J^* for the optimal controller and the simplified controller. After the gains have been differenced in the main program, the remainder of the run is accomplished by successive calls to the COV and PRB subroutines.

Comments on Future Use

The current version of the optimization program was written in FORTRAN IV and run on the CDC 6600 computer. It is my understanding that future runs will be made on an IBM 7094 computer which has a 32K memory. The CDC 6600 computer that was used for the current version of the program has a 65K memory. The optimization program requires approximately 33K of memory, 22K of which is used for arrays and 11K for the program itself. Obviously, if the program is to be run on a 32K computer some modifications must be made.

Modifications for 32K Computer

It is suggested that the current program be divided into two programs made up of the following subroutines.†

PROGRAM 1††	MAIN PROGRAM
	START SUBROUTINE
	QOEF SUBROUTINE
	GAIN SUBROUTINE
PROGRAM 2	MAIN PROGRAM
	START SUBROUTINE
	COEF SUBROUTINE
	COV SUBROUTINE
	PRB SUBROUTINE
	DRV SUBROUTINE
	REND SUBROUTINE
	PAR SUBROUTINE
	END SUBROUTINE

Data for Modified Programs

The data deck for program 1 will be the same as the data deck used for an "optimization iteration" run, (see figure 1) with the card representing the value of IGAIN omitted. The only subroutine called from the main program is the GAIN subroutine. The results of a run with program 1 will be a set of gains K(t) and deterministic input f(t), (for t=0,1,2,...,159,160 seconds) in the form of punched

† This procedure worked during the debugging stage when the optimization program was run on the H-1800 which is a 32K computer.

†† The arrays RR and DP have been removed from dimension and common statements in Program 1.

cards as described in Section III - OUTPUT.

The data deck for program 2 will be the same as the data deck used for a simplification run (see figure 2) with the card representing the value of IGAIN omitted. The results of a run with program 2 will be the response means and covariances, the cost J^* and the partial derivatives $\frac{\partial f}{\partial R}$ and $\frac{\partial f}{\partial S}$.

One iteration in the optimization process then, would require two computer runs. The first, with program 1 to obtain a controller for a particular set of quadratic weights. The second with program 2 to obtain responses, cost functional and derivatives for the controller calculated by program 1.

Program Running Time

The major part of the total running time for the optimization program occurs in the GAIN subroutine and the COV subroutine (i.e. approximately 95% of the total run time is used to compute gains and response means and covariances). One optimization iteration run on the CDC 6600 takes approximately 21.5 minutes. An identical run on the Honeywell H-1800 takes approximately 210 minutes. The gains calculations takes 10.5 minutes on the CDC 6600 and 105 minutes on the H-1800. The response means and covariance calculation takes 10 minutes on the CDC 6600 and 97 minutes on the H-1800. The above run times reflect the 10 to 1 ratio of the floating multiply operation for the two computers (i.e. the CDC 6600 has a 1 microsecond floating multiply and the H-1800 has a 10 microsecond floating multiply). This is not surprising since the time required for both the gains calculations and the response means and covariance calculations is due almost entirely to the matrix multiplications required by the respective difference equations. According to the best estimate currently available, the IBM 7094 MOD II computer runs 25% faster than the H-1800. On the basis of this estimate, it is expected that an optimization iteration run on the IBM 7094 would take approximately 160 minutes. The run with program 1 (gains calculations) would take approximately 85 minutes and the run with program 2 (response means and covariances and derivatives) would take approximately 75 minutes.

APPENDIX A

PROGRAM FLOW CHARTS

A functional flow chart of the optimization program in addition to more detailed flow charts of the main program and each of the subroutine subprograms are presented in this appendix as figures A-1 through A-12.

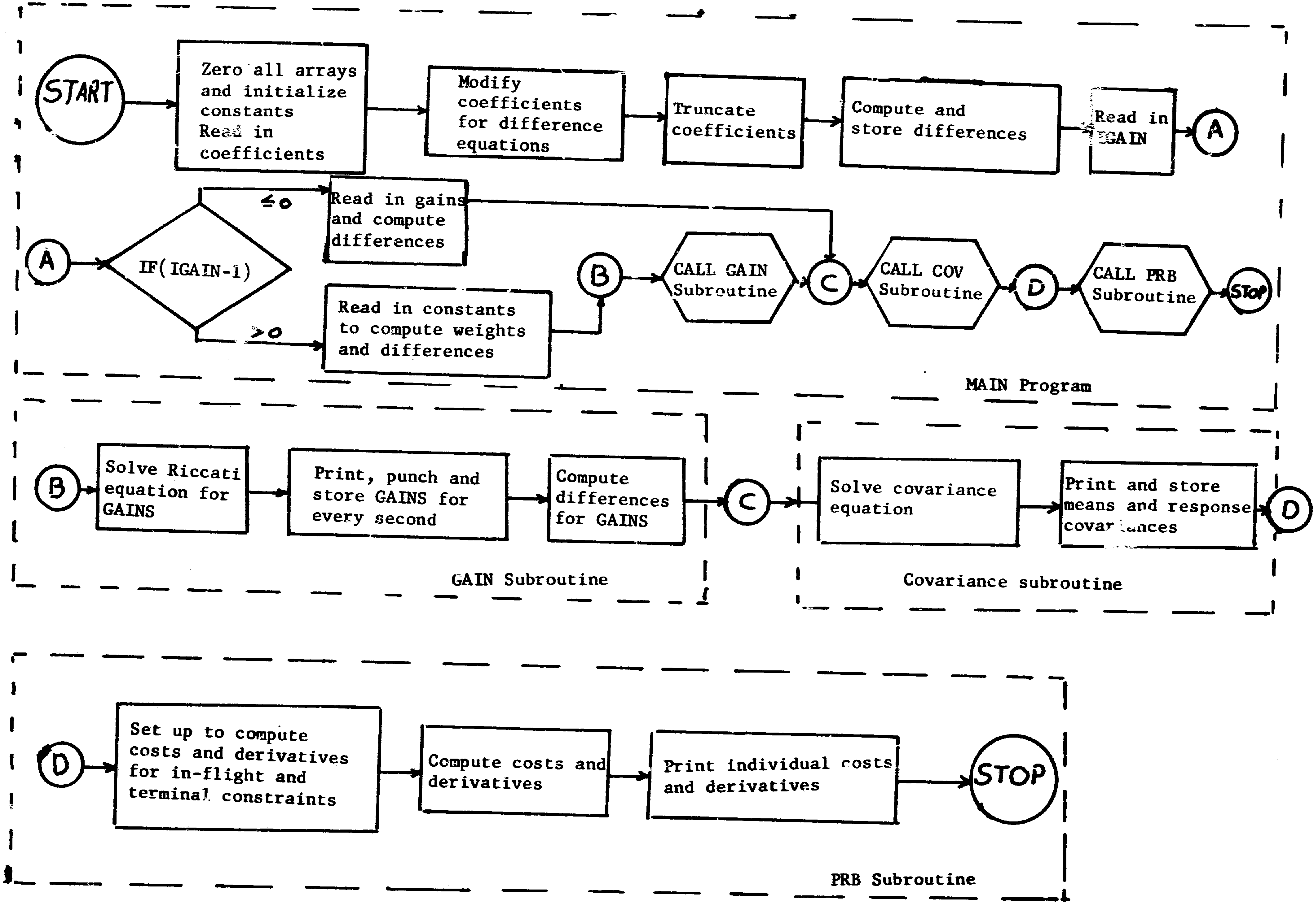

FIGURE A-1 FUNCTIONAL FLOW CHART OF OPTIMIZATION PROGRAM

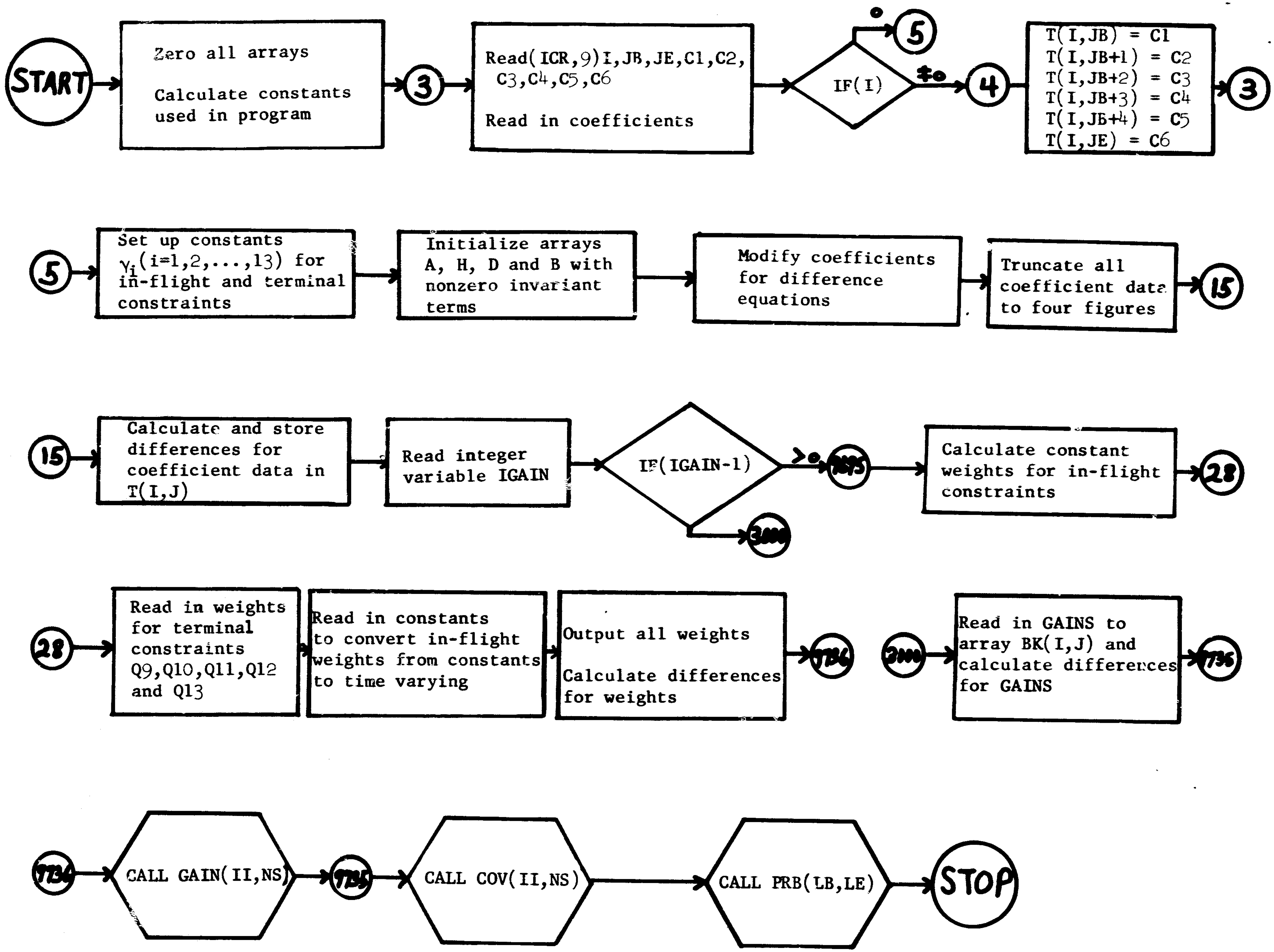

FIGURE A-2 DETAILED FLOW CHART MAIN PROGRAM

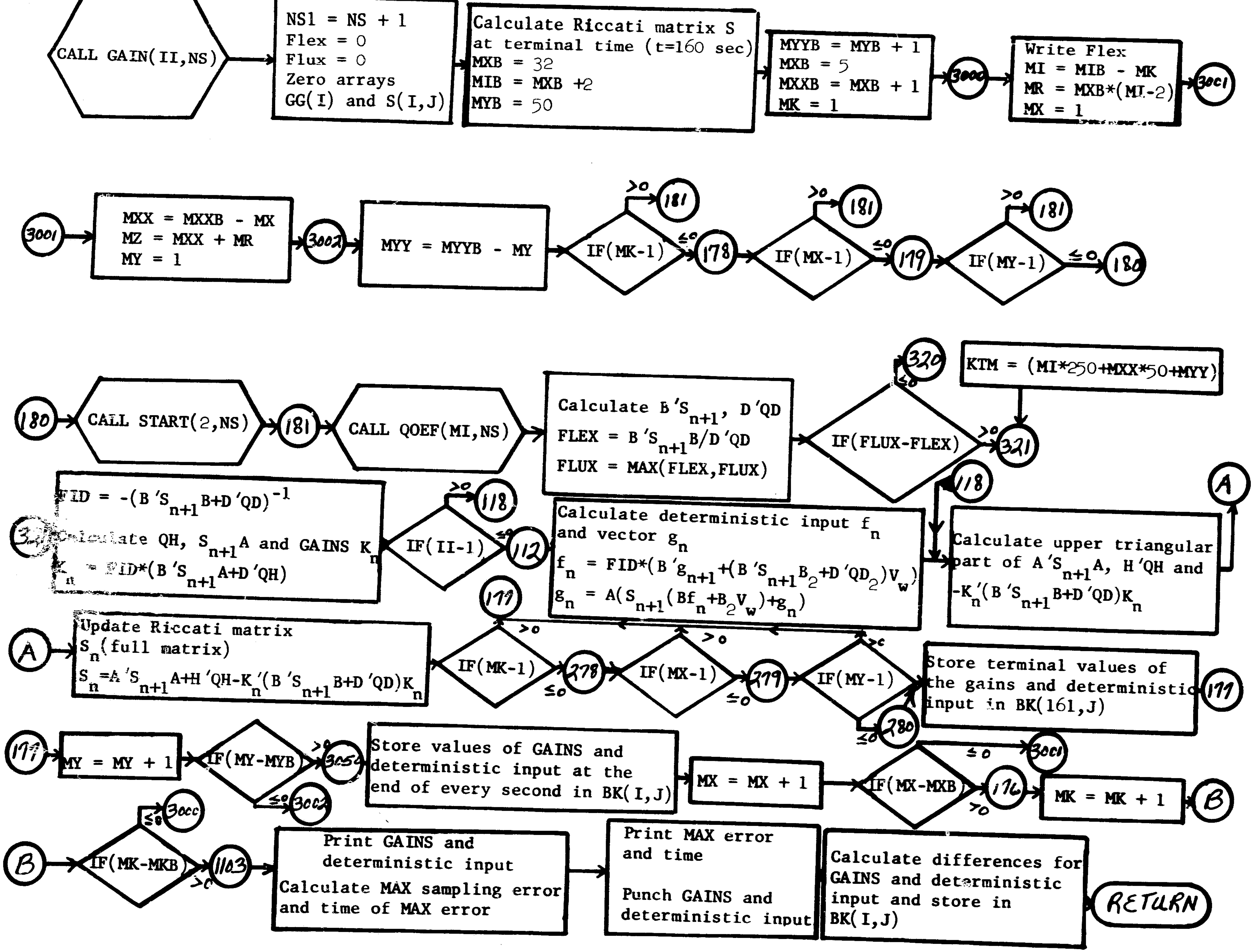

FIGURE A-3 DETAILED FLOW CHART OF GAIN SUBROUTINE

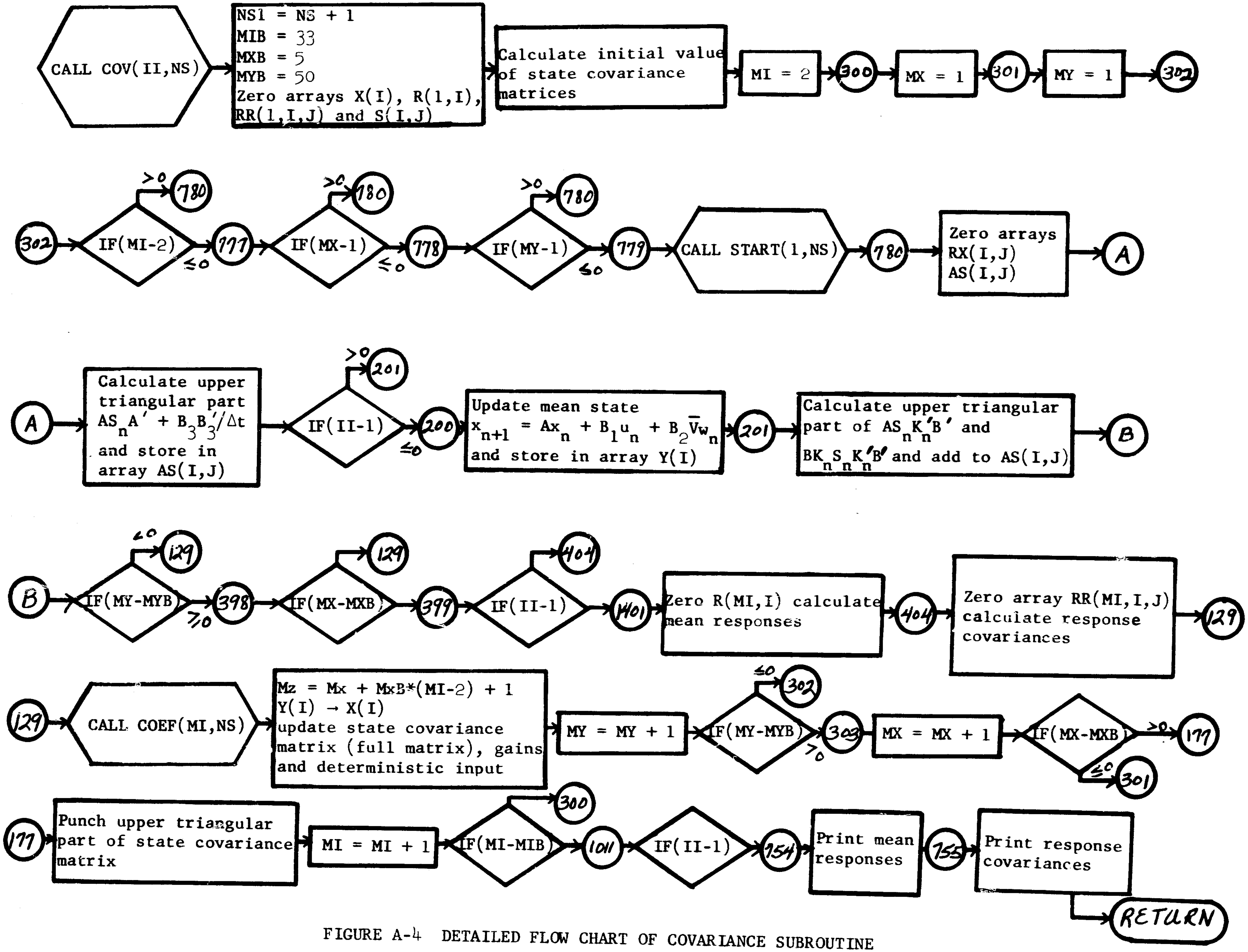

FIGURE A-4 DETAILED FLOW CHART OF COVARIANCE SUBROUTINE

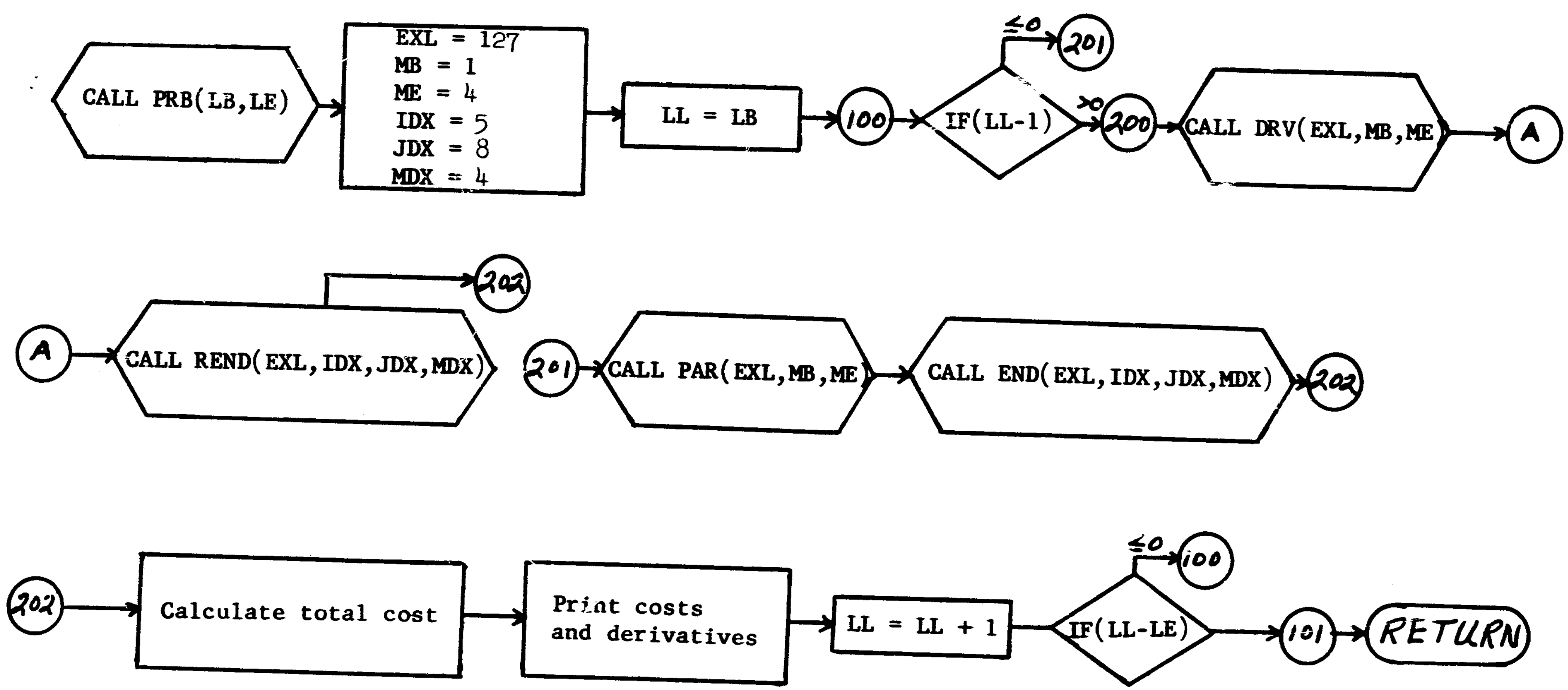

FIGURE A-5 DETAILED FLOW CHART OF PRB SUBROUTINE

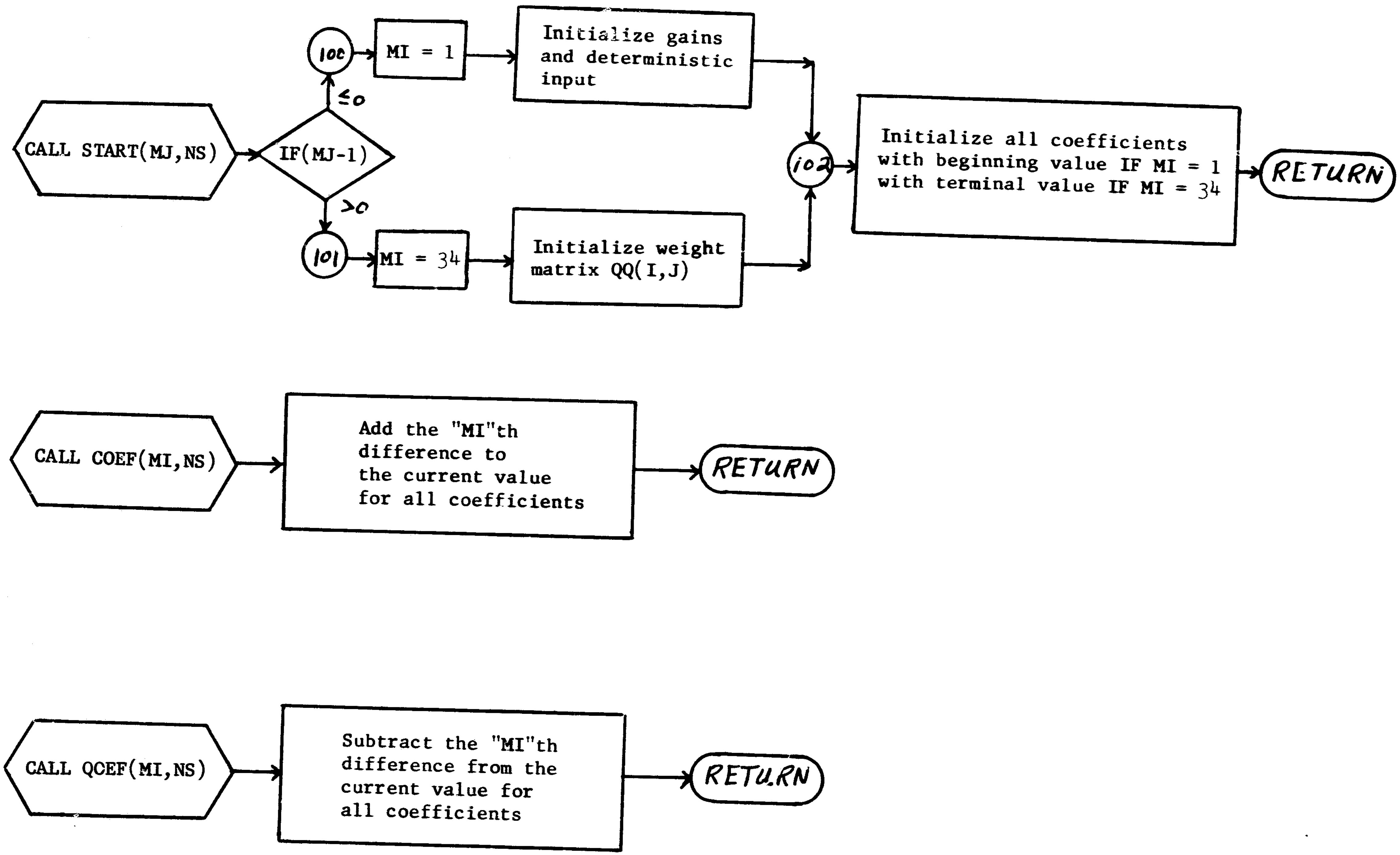

FIGURE A-6 FLOW CHART OF START, COEF, QOEF SUBROUTINES

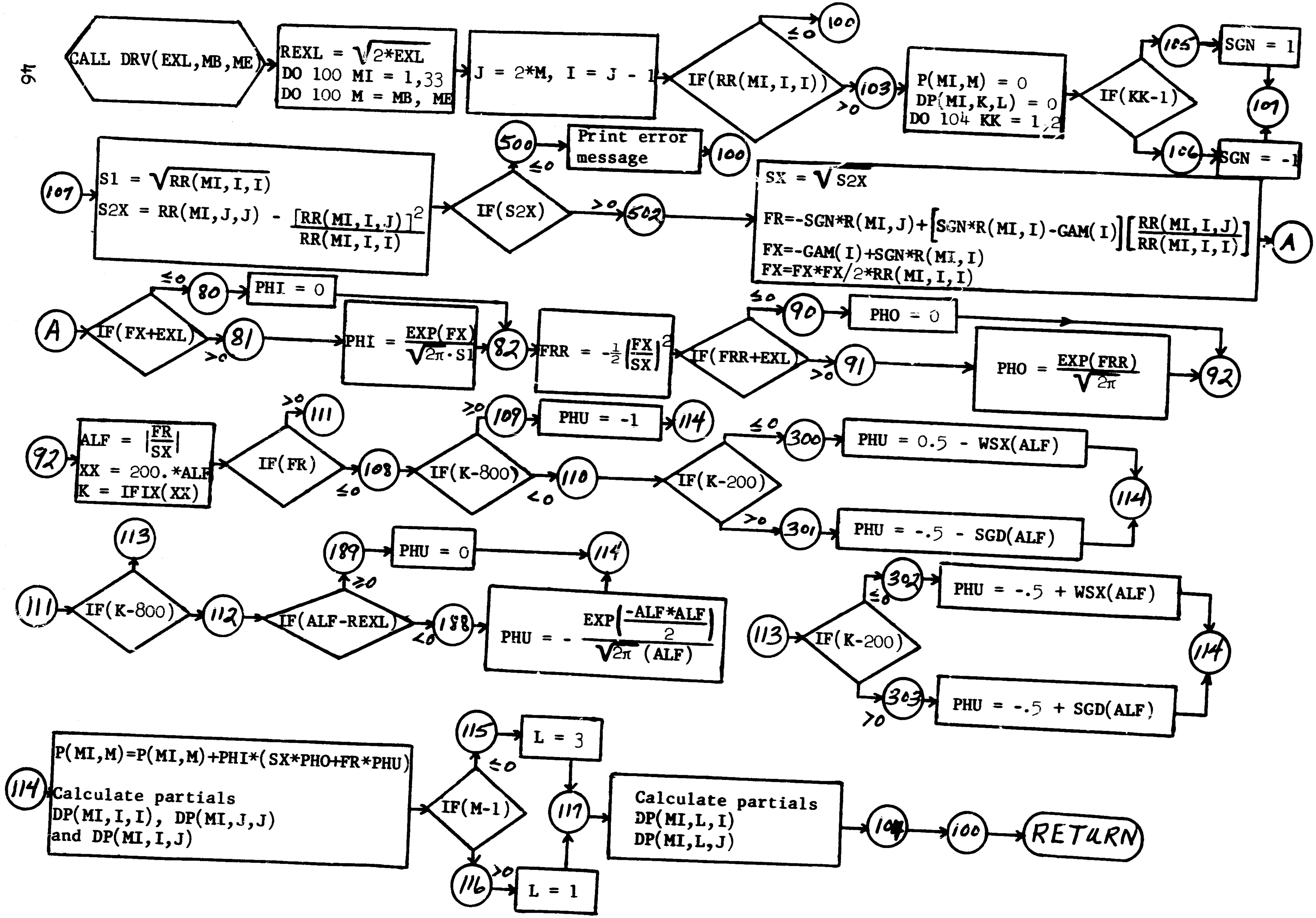

FIGURE A-7 FLOW CHART OF DRV SUBROUTINE

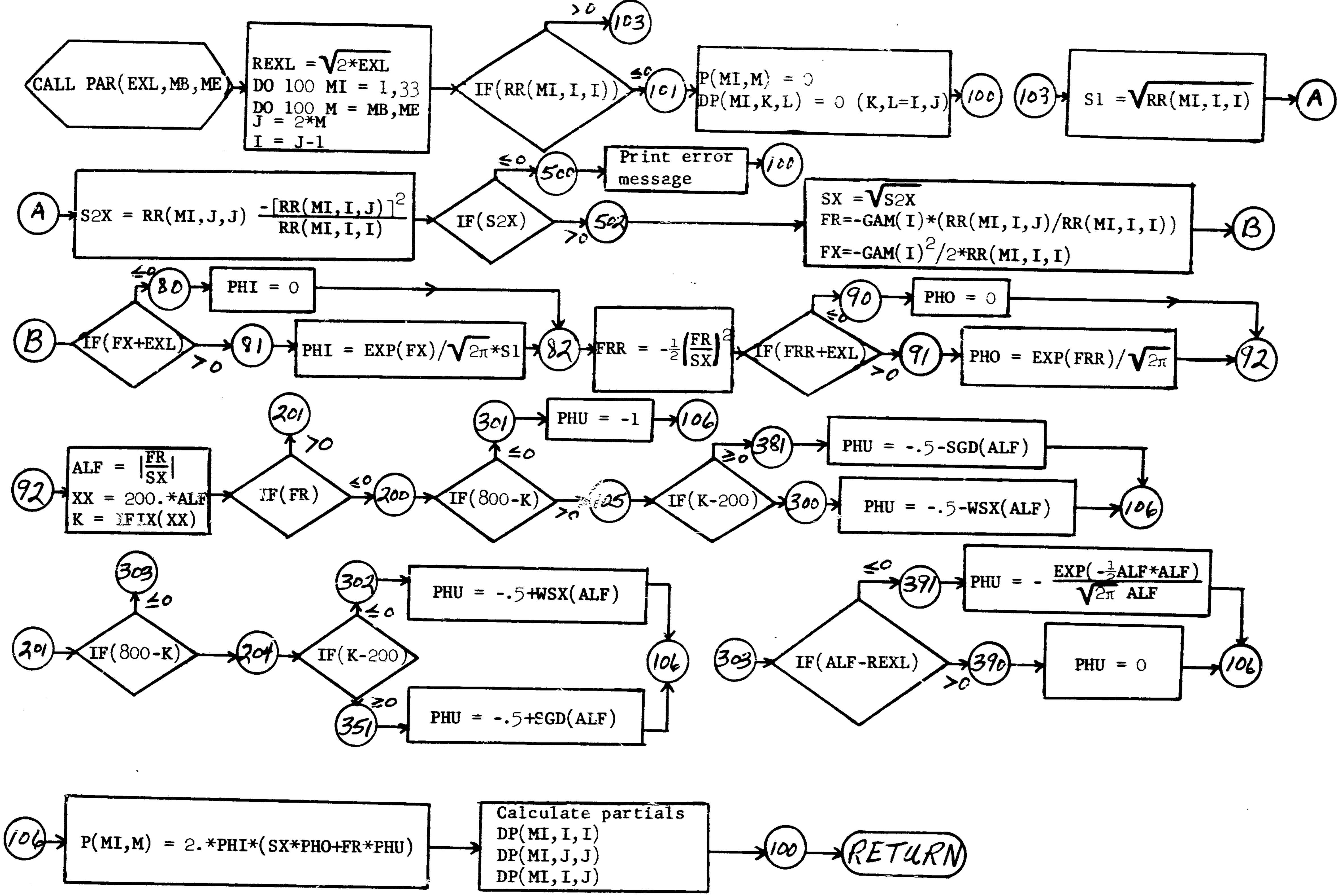

FIGURE A-8 FLOW CHART OF PAR SUBROUTINE

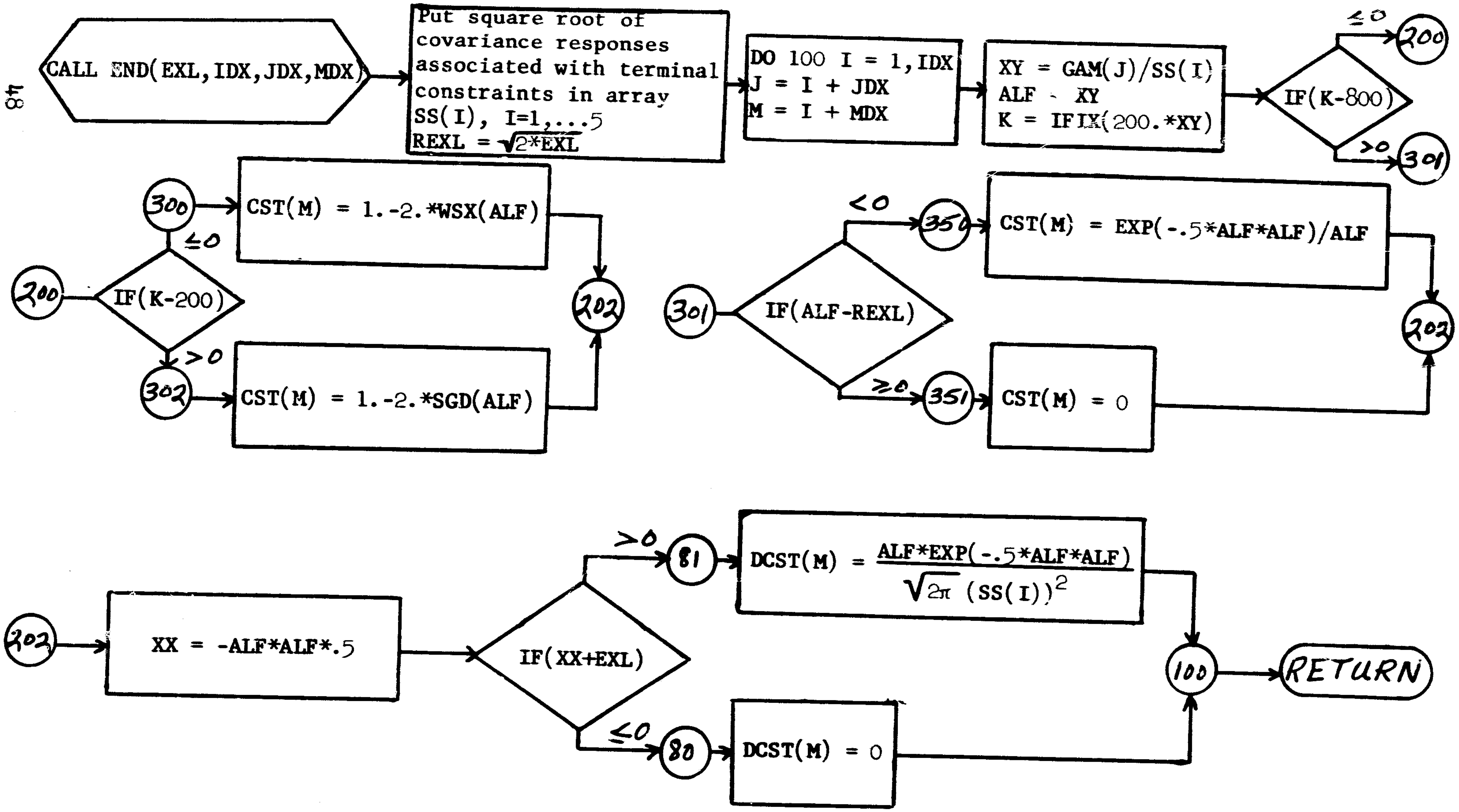

FIGURE A-9 FLOW CHART OF END SUBROUTINE

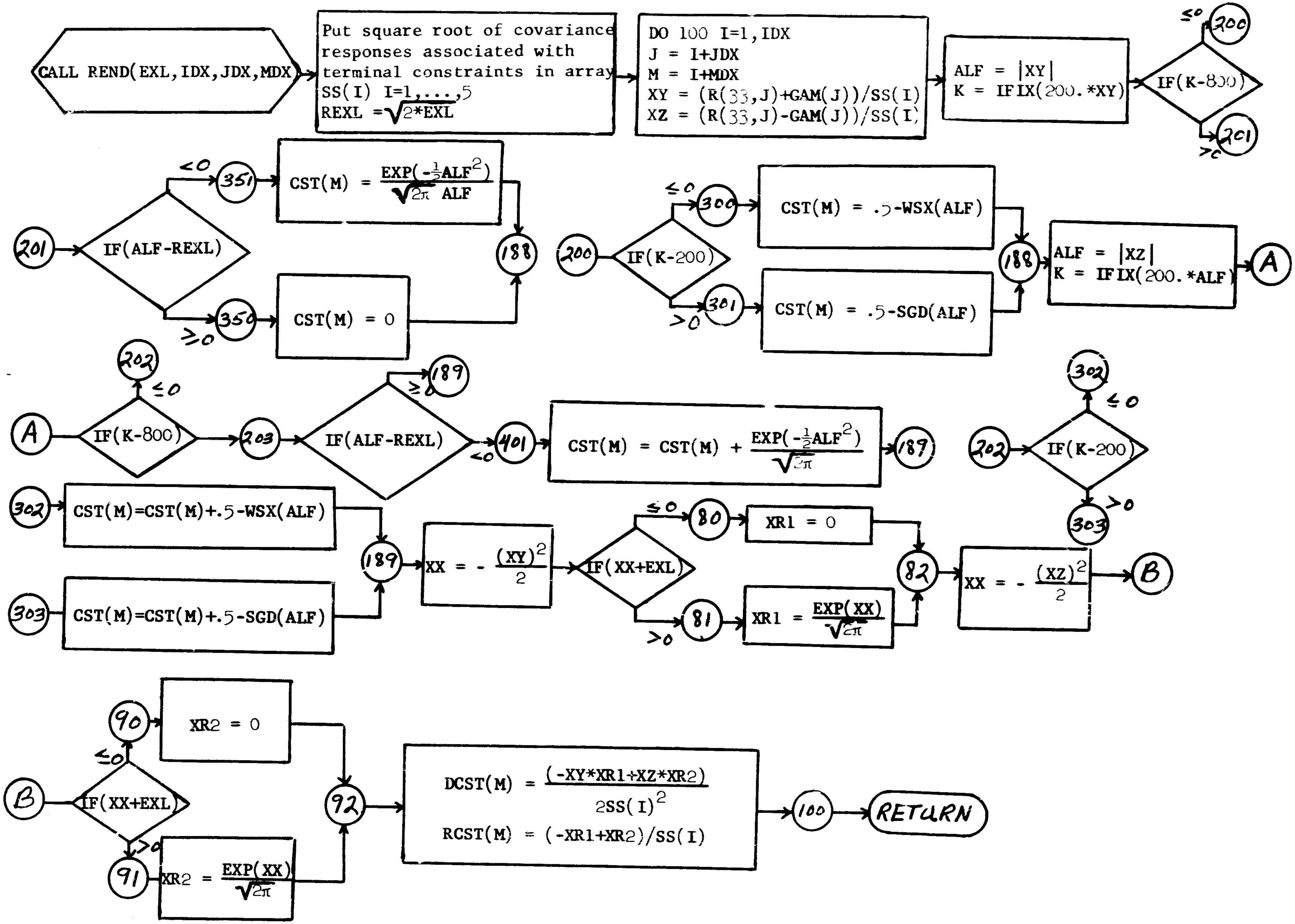

FIGURE A-10 FLOW CHART OF REND SUBROUTINE

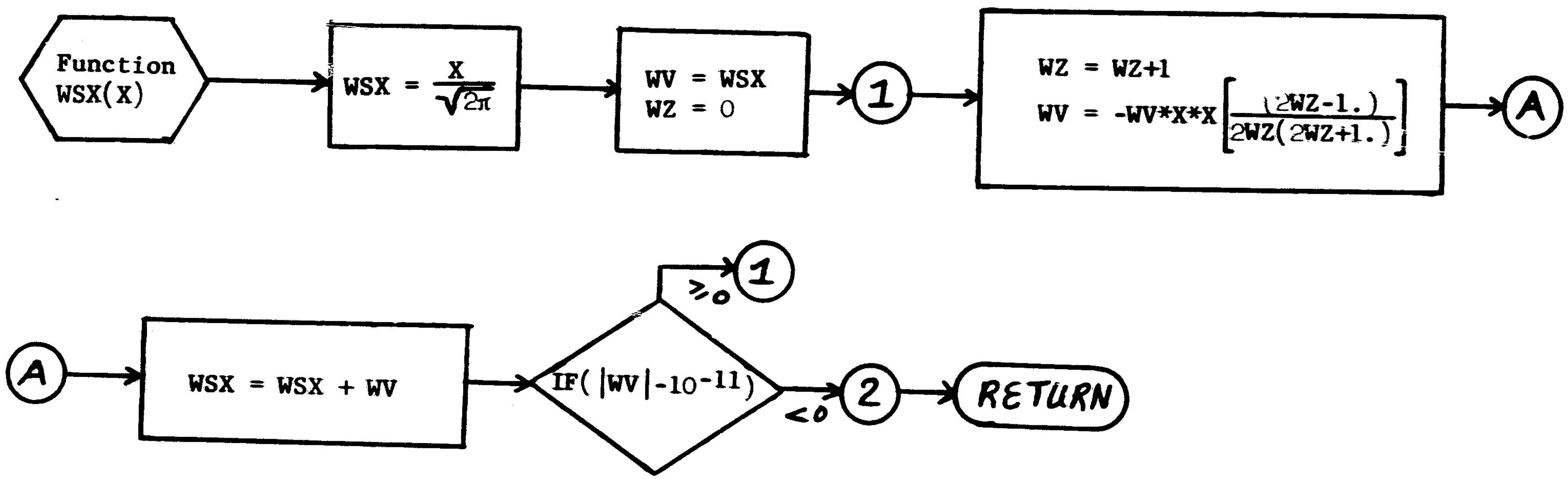

FIGURE A-11 FLOW CHART OF FUNCTION SUBROUTINE WSX(x)

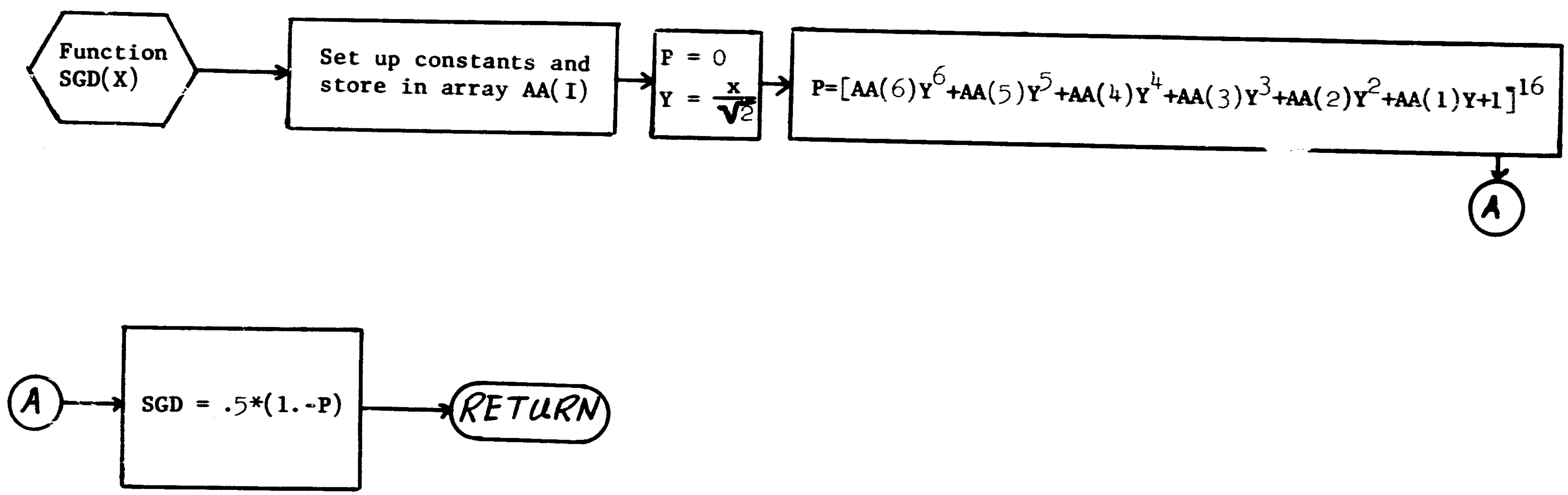

FIGURE A-12 FLOW CHART OF THE FUNCTION SUBROUTINE SGD(x)

APPENDIX B

"DATA ARRAY" T(I,J)

The elements of the matrices $A(n)$ and $H_1(n)$ and the components of the vectors $B_1(n)$, $B_2(n)$, $B_3(n)$, $D_1(n)$ and $D_2(n)$ which appear in the difference equation for the state

$$x(n+1) = A(n)x(n) + B_1(n)u(n) + B_2(n)\bar{v}_\omega(n) + B_3(n)\eta(n),$$

and the response vector equation

$$r(n) = H_1(n)x(n) + D_1(n)u(n) + D_2(n)\bar{v}_\omega(n)$$

are associated with the column index J of the "data array" T(I,J) as explained in Section II - MAIN PROGRAM. This appendix presents graphically where the elements and components are stored according to the index J. Circled numbers indicate constants, uncircled numbers indicate the index J and blank indicates zero values. Constant and zero values are computed once and for all in the beginning of the main program.

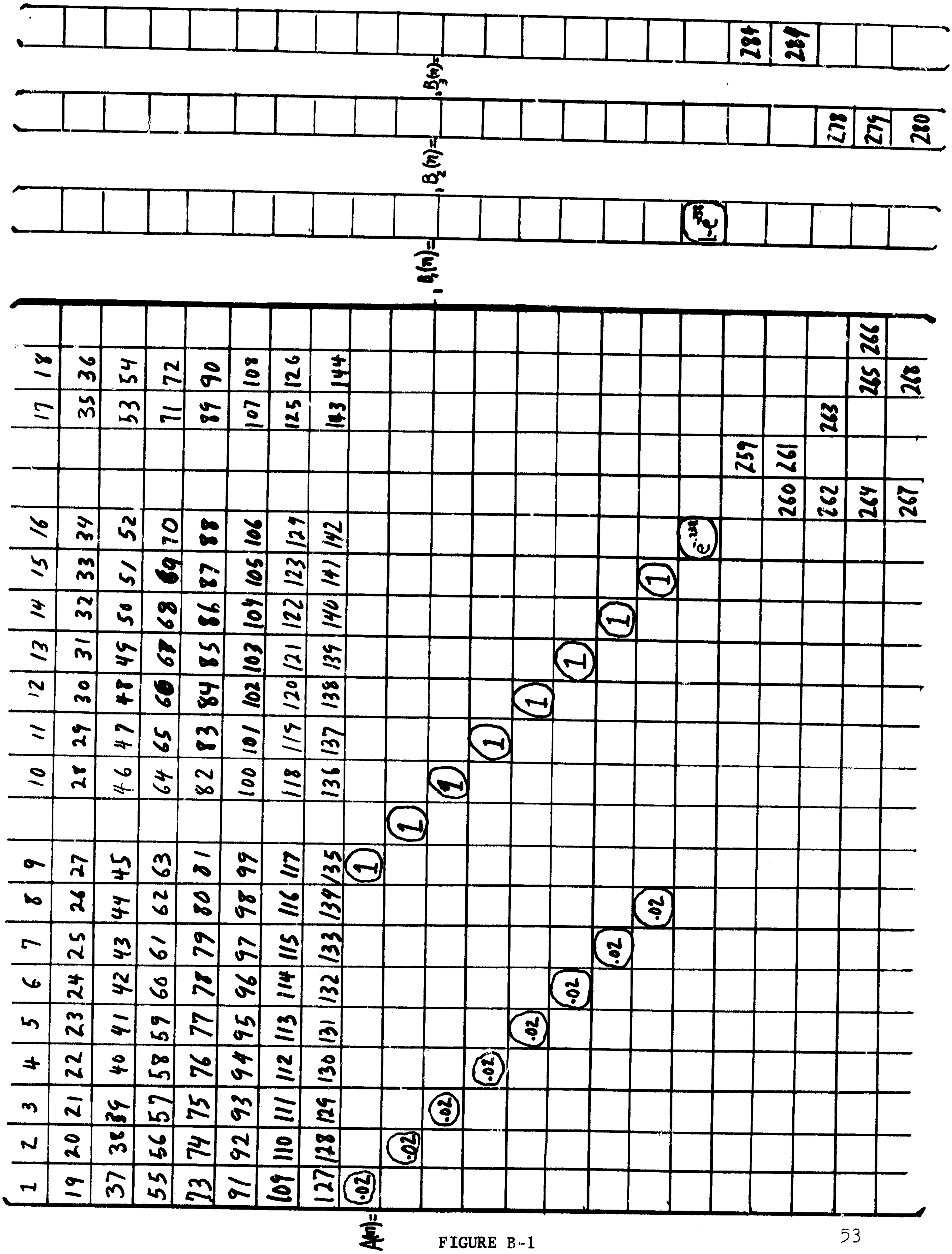

FIGURE B-1

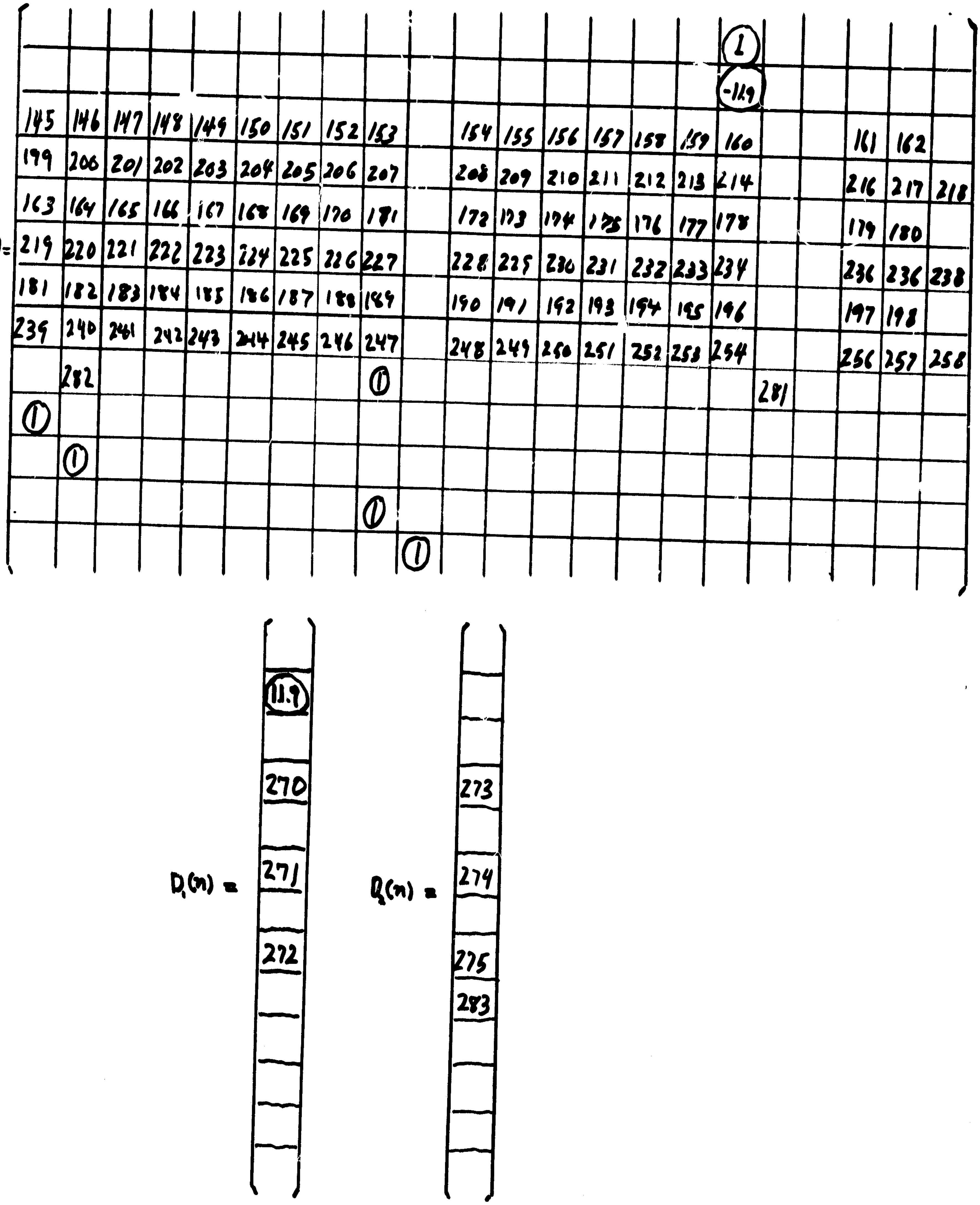
1
-11.9
145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162
199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 216 217 218
163 164 165 166 167 168 169 170 181 172 173 174 175 176 177 178 179 180
$H_1(n)=$
219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 236 236 238
181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198
239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 256 257 258
282 1 281
1
1
1
1
$D_1(n) =$
11.9
270
271
272
$D_2(n) =$
273
274
275
283

FIGURE B-2

APPENDIX C

FORTRAN NOTATION

This appendix equates the mathematical notation in references 1 and 2 with the corresponding FORTRAN notation used in the optimization program.

MATHEMATICAL NOTATION	DEFINITION	FORTRAN NOTATION
$A(n)$	Coefficient matrix for the state vector in the difference equation for the state.	A(I,J)
$B_1(n)$	Vector multiplied by the control input $u(n)$ in the difference equation for the state	B(I)
$B_2(n)$	Vector multiplied by mean incident wind $\bar{v}_\omega(n)$ in the difference equation for the state	B2(I)
$B_3(n)$	Vector multiplied by white noise input $\eta(n)$ in the difference equation for the state	G(I)
$H_1(n)$	Coefficient matrix for the state vector in the equation for the response vector $r(n)$	H(I,J)
$D_1(n)$	Vector multiplied by the control input $u(n)$ in the equation for the response vector $r(n)$	D(I)
$D_2(n)$	Vector multiplied by the mean incident wind $\bar{v}_\omega(n)$ in the equation for the response vector $r(n)$	D2(I)
$\bar{v}_\omega(n)$	Mean incident wind	VW
$f(n)$	Deterministic input	F
$K(n)$	Optimal gains vector	AK(I)
$P(n)$	Updated Riccati Matrix	AS(I,J)
$P(n+1)$	Current Riccati Matrix	S(I,J)
$Q(n)$	Quadratic weight matrix	QQ(I,J)
$g(n)$	Updated value of $g(n+1)$	GH(I)

MATHEMATICAL NOTATION	DEFINITION	FORTRAN NOTATION
$g(n+1)$	Vector used to calculate f(n)	GG(I)
$X(n)$	Current value of state covariance matrix	S(I,J)
$X(n+1)$	Update value of state covariance matrix	AS(I,J)
$S(n)$	Response covariance matrix	RR(I,J,K)
$\bar{r}(n)$	Mean response vector	R(I,J)
γ_i	Constants which represent the terminal and in-flight limits for constrained responses	GAM(I)
J^*	Cost functional to be minimized	TCST
$P(\bar{a}_i)$	Terminal likelihoods for α, $\dot{\phi}$, $\dot{z}$, ϕ, z	CST(I) (for I=5,6,7,8,9)
$E\{N_i\}$	In-flight expectations for β, I_{B1}, I_{B2} and I_{B3}	CST(I) (for I=1,2,3,4)
$\frac{\partial}{\partial t} E\{N_i\}$	In-flight expectation densities for β, I_{B1}, I_{B2} and I_{B3}	P(I,J)
$\left.\begin{matrix}\frac{\partial f_1}{\partial S_{i,i}} \\ \frac{\partial f_1}{\partial \bar{r}_i}\end{matrix}\right\}$	Partials of the terminal likelihoods with respect to the response covariances and mean responses	DCST(I) (for I=5,6,7,8,9) RCST(I) (for I=5,6,7,8,9)
$\left.\begin{matrix}\frac{\partial f_2}{\partial S_{ij}} \\ \frac{\partial f_2}{\partial R_{i,i}}\end{matrix}\right\}$	Partials of the in-flight expectations with respect to the response covariances and mean responses	DP(I,J,K) DP(I,J,K)
$\bar{x}(n)$	Current mean state vector	X(I)
$\bar{x}(n+1)$	Updated mean state vector	Y(I)
$u(n)$	Control input to the gimbal actuator	FIV and FIE

MATHEMATICAL NOTATION	DEFINITION	FORTRAN NOTATION
C_1	Constants in wind filter equations	DD1
C_2		DD2
C_3		AL(1)
C_4		AL(2)
C_5		1
Δt	Sample time used in difference equation approximation	DT

REFERENCES

1. L. D. Edinger, et al, "Design of a Load-Relief Control System", NASA Contractor Report CR-61169, April 21, 1967.

2. C. A. Harvey, "Application of Optimal Control Theory to Launch Vehicles", NAS8-21063.